# CORE 2 Summary Notes

## 1 Indices

- Rules of Indices

$x^a \times x^b = x^{a+b}$

$x^a \div x^b = x^{a-b}$

$(x^a)^b = x^{ab}$

- $x^0 = 1$
- $x^{-a} = \frac{1}{x^a}$
- $x^{\frac{1}{n}} = \sqrt[n]{x}$
- $x^{\frac{m}{n}} = (\sqrt[n]{x})^m = \sqrt[n]{x^m}$

Solve $9^x = \frac{1}{27}$

$3^{2x} = 3^{-3}$

$2x = -3$

$x = -\frac{3}{2}$

Solve $3^{2x+1} - 10 \times 3^x + 3 = 0$

$3 \times (3x)^2 - 10 \times 3^x + 3 = 0$

Let $y = 3^x$

$3y^2 - 10y + 3 = 0$

$(3y - 1)(y - 3) = 0$

$y = \frac{1}{3}$ gives $3^x = \frac{1}{3}$ $\quad x = -1$

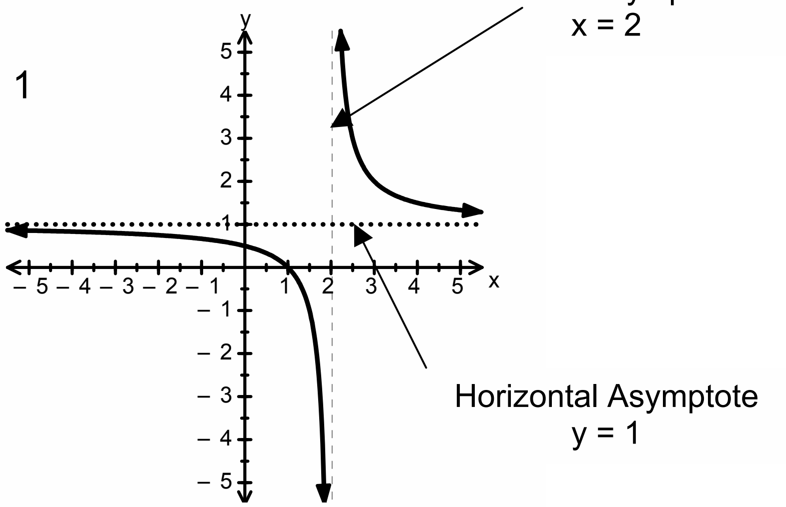
$y = 3$ gives $3^x = 3$ $\quad x = 1$

## 2 Graphs and Transformations

ASYMPTOTES

Straight lines that are approached by a graph which never actually meets them.

$$y = \frac{1}{x-2} + 1$$




TRANSLATION - to find the equation of a graph after a translation of $\begin{bmatrix} a \\ b \end{bmatrix}$ you replace x by (x-a) and y by (y - b)

e.g. The graph of $y = x^2 - 1$ is translated through $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Write down the equation of the graph formed.

$(y + 2) = (x-3)^2 - 1$

$y = x^2 - 6x + 6$

| |
|---|
| y - b = f(x-a)<br><br>or<br><br>y = f(x-a) +b |

$y = x^2 - 1$

$y = x^2 - 6x + 6$

REFLECTING

Reflection in the x-axis, replace y with –y ← $y = -f(x)$

Reflection in the y-axis, replace x with – x ← $y = f(-x)$

STRETCHING

Stretch of factor k in the x direction replace x by $\frac{1}{k}x$ ← $y = f(\frac{1}{k}x)$

Stretch of factor k in the y direction replace y by $\frac{1}{k}y$ ← $y = kf(x)$

e.g. Describe a stretch that will transform $y = x^2 + x - 1$ to the graph of $y = 4x^2 + 2x - 1$

$4x^2 + 2x - 1 = (2x)^2 + (2x) - 1$
So x has been replaced by 2x.

Stretch of scale factor ½ in the x direction

## 3 Sequences and Series 1

- A sequence can be defined by the nth term such as $u_n = n^2 + 1$

  $u_1 = 1^2 + 1 = 2$ $\quad u_2 = 2^2 + 1 = 5$ $\quad u_3 = 3^2 + 1 = 10$ $\quad u_4 = 4^2 + 1 = 17$

- An INDUCTIVE definition defines a sequence by giving the first term and a rule to find the next terms.

  $U_{n+1} = 2u_n + 1 \quad u_1 = 3 \quad u_2 = 7 \quad u_3 = 15$

- Some sequences get closer and closer to a value called the LIMIT – these are known as CONVERGING sequences

  *e.g The sequence defined by* $U_{n+1} = 0.2u_n + 2 \quad u_1 = 3$ *converges to a limit l*
  *Find the value of l.*

  *l must satisfy the equation* $l = 0.2l + 2$

  $0.8l = 2$

  $l = 2 \div 0.8$

  $= 2.5$

ARITHMETIC SEQUENCE

Each term is found by adding a fixed number (COMMON DIFFERENCE) to the previous one.

$u_1 = a \quad u_2 = u_1 + d \quad u_3 = u_2 + d$

a is the first term ,d is the common difference the sequence is
a, a + d , a + 2d, a + 3d,……….

$U_n = a + (n - 1)d$

SUM of the first n terms of an AP (Arithmetic progression)

$S_n = \frac{1}{2}n(2a + (n-1)d)$ or $\frac{1}{2}n(a + l)$ where l is the last term

- The sum of the first n positive integers is $\frac{1}{2}n(n + 1)$

- SIGMA notation $\sum_{i=0}^{5} i^3 = 0^3 + 1^3 + 2^3 + 3^3 + 4^3 + 5^3$

Show that $\sum_{r=1}^{20} (3n - 1) = 610$

$= 2 + 5 + 8 + .\cdots\cdots 59$

$a = 2 \quad d = 3$

$S_{20} = \frac{20}{2}(2 \times 2 + (20 - 1) \times 3)$

$= 10 \times 61$

$= 610$

## 4 Geometric Sequences

- A geometric sequence is one where each term is found by MULTIPLYING the previous term by a fixed number (COMMON RATIO)
- The nth term of a geometric sequence a, ar, $ar^2$, $ar^3$, …is $ar^{n-1}$
  **a** is the first term **r** is the common ratio
- The sum of a geometric sequence a + ar + $ar^2$ + $ar^3$ ….+ $ar^{n-1}$ is a geometric series $S_n = \frac{a(r^n - 1)}{r - 1}$
- If -1 < r < 1 then the sum to infinity is $\frac{a}{1-r}$

Find the sum to inifinty of the series $1 + \frac{2}{3} + \frac{4}{9} + \frac{8}{27}$

Geometric Series first term = 1 common ratio = $\frac{2}{3}$

Sum to infinity = $\frac{1}{1-\frac{2}{3}} = 3$

## 5 Binomial Expansion

- The number of ways of arranging n objects of which r are one type and (n-r) are another is given by $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ where n! = n(n-1)(n-2)….x 3 x 2 x 1
- $(1 + x)^n = 1 + \binom{n}{1} x + \binom{n}{2} x^2 + \binom{n}{3} x^3 + .\cdots\cdots x^n$

$(a + b)^n = a^n + \binom{n}{1} a^{n-1}b^1 + \binom{n}{2} a^{n-2}b^2 + .\cdots\cdots. \binom{n}{n-1} a^1 b^{n-1} + b^n$

Find the coefficient of $x^3$ in the expansion of $(2+3x)^9$

*Note 3 + 6 = 9 (n)*

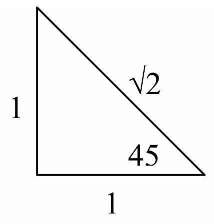
n = 9 , r = 3 $\binom{9}{3}(3x)^3(2)^6$

$= 84 \times 27 x^3 \times 64 = 145152 x^3$

## 6 Trigonometry

- **Exact Values – LEARN**

**45**




$\sin 45 = \frac{1}{\sqrt{2}}$

$\cos 45 = \frac{1}{\sqrt{2}}$

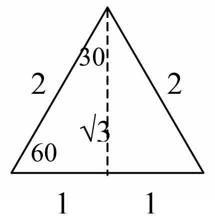
$\tan 45 = 1$

**30**

$\sin 30 = \frac{1}{2}$

$\cos 30 = \frac{\sqrt{3}}{2}$

$\tan 30 = \frac{1}{\sqrt{3}}$




**60**

$\sin 60 = \frac{\sqrt{3}}{2}$

$\cos 60 = \frac{1}{2}$

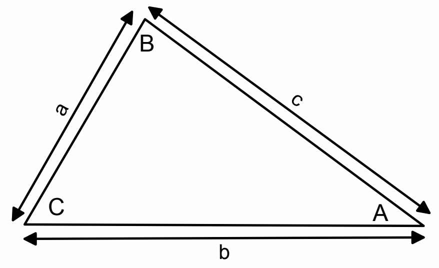
$\tan 60 = \sqrt{3}$

COSINE RULE

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos A$$



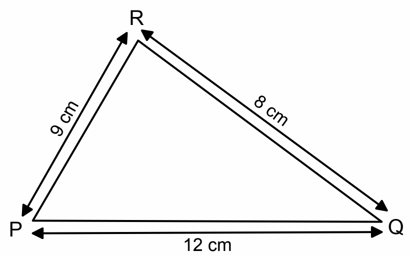

SINE RULE

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

AREA of a TRIANGLE

$$\frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B$$

e.g. Find the area of triangle PQR




*Finding angle P*

$$8^2 = 9^2 + 12^2 - 2 \times 9 \times 12 \times \cos P$$

$$\cos P = \frac{9^2 + 12^2 - 8^2}{2 \times 9 \times 12} = 0{\cdot}745$$

$$P = 41{\cdot}8\cdots\cdots.$$

$$Area = \frac{1}{2} \times 9 \times 12 \times \sin 41{\cdot}8\cdots\cdots.$$

$$= 35{\cdot}99991\,36\ cm^2 =$$

$$= 36\ cm^2$$

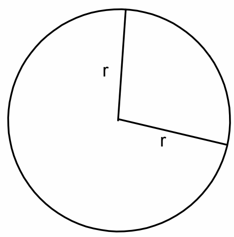
*Try to avoid rounding until you reach your final answer*

RADIANS and ARCS

$360^{\circ} = 2\pi$ radians

$180 = \pi$ radians $\quad 90 = \frac{\pi}{2}$ radians $\quad 60 = \frac{\pi}{3}$ radians

$45 = \frac{\pi}{4}$ radians $\quad 30 = \frac{\pi}{6}$ radians

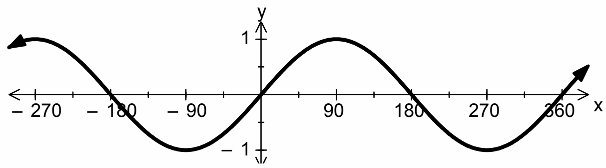



LENGTH OF AN ARC $l = r\theta$

AREA of SECTOR = $\frac{1}{2} r^2\theta$

## 7 Trigonometry

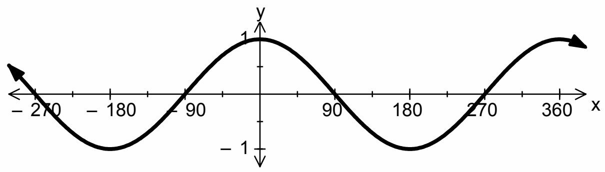
$y = \sin\theta$




$\sin(-\theta) = -\sin\theta$ $\sin(180 - \theta) = \sin\theta$

$\sin(\pi - \theta) = \sin\theta$ (in radians)

$y = \cos\theta$

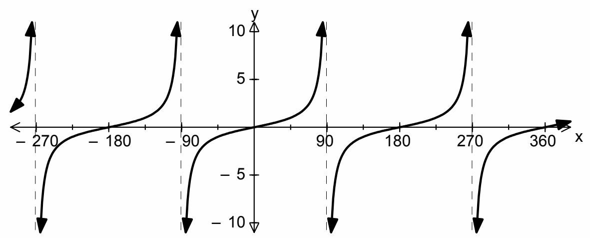



$\cos(-\theta) = \cos\theta$ $\cos(180 - \theta) = -\cos\theta$

$\cos(\pi - \theta) = -\cos\theta$ (in radians)

$y = \tan\theta$

$\tan x = \dfrac{\sin x}{\cos x}$




$\tan(-\theta) = -\tan\theta$

<u>SOLVING EQUATIONS</u> $\sin^2 x + \cos x^2 = 1$

LEARN THIS IDENTITY

e.g. Solve the equation $2\sin^2 x = 3\cos x$ for $0 < x < 2\pi$

$$2\sin^2 x = 3\cos x$$

$$2(1 - \cos^2 x) = 3\cos x$$

$$2\cos^2 x + 3\cos x - 2 = 0$$

$$(2\cos x - 1)(\cos x + 2) = 0$$

$\cos x = -2$ has no solution

cos x = -½ $\quad x = \frac{\pi}{3}$ or $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

PROVING IDENTITIES

Show that $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x$

*LHS*: $(\sin x + \cos x)^2 = (\sin^2 x + 2\sin x \cos x + \cos^2 x)$

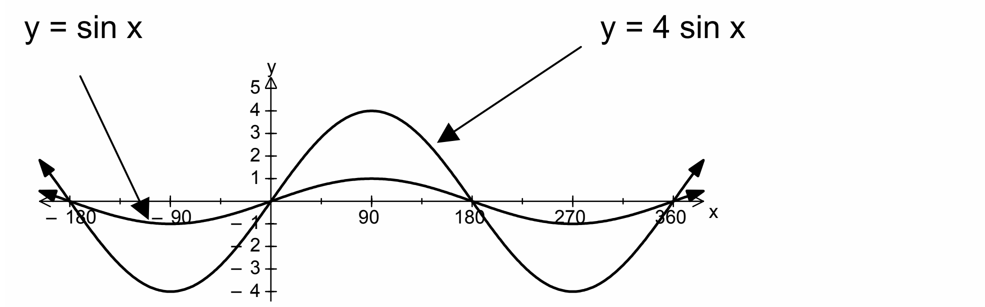
$= \sin^2 x + \cos^2 x + 2\sin x \cos x$

$= 1 + 2\sin x \cos x$

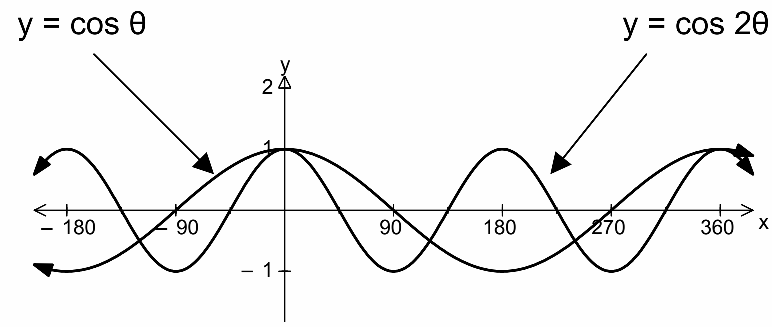
LHS = RHS

TRANSFORMATIONS

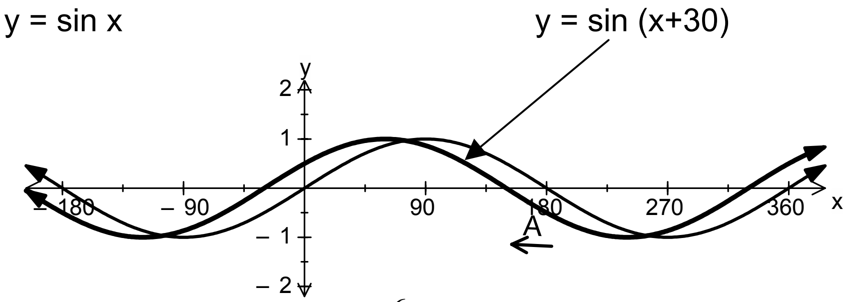
Stretch in the **y direction** scale factor a $\quad y = a \sin \theta$



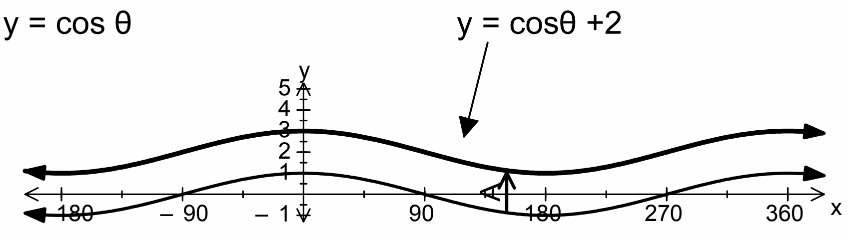

Stretch in the **θ direction** scale factor $\frac{1}{b}$ $\quad y = \cos b\theta$




Translation $\begin{bmatrix} -c \\ 0 \end{bmatrix}$ $\quad y = \sin(\theta + c)$

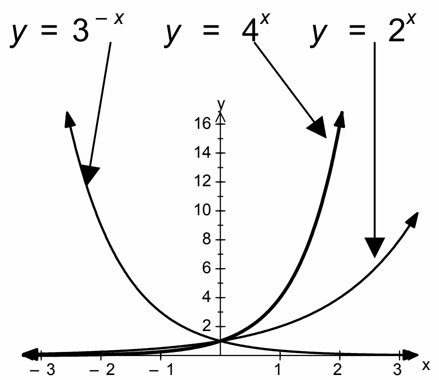


Translation $\begin{bmatrix} 0 \\ d \end{bmatrix}$ $y = \cos\theta + d$




## 8 Exponentials and Logarithms

- A function of the form $y = a^x$ is an exponential function
- The graph of $y = a^x$ is positive for all values of x and passes through the point (0,1)




- A Logarithm is the inverse of an exponential function

$$y = a^x \Rightarrow x = \log_a y$$

e.g $\log_2 32 = 5$ because $2^5 = 32$

LEARN THE FOLLOWING

$\log_a a = 1$ $\qquad \log_a 1 = 0$

$\log_a a^x = x$ $\qquad a^{\log x} = x$

Laws of Logarithms

$$\log_a m + \log_a n = \log_a mn$$

$$\log_a m - \log_a n = \log_a \left(\frac{m}{n}\right)$$

$$k\log_a m = \log_a m^k$$

E.g Solve $\log_x 4 + \log_x 16 = 3$

$$\log_x 64 = 3$$

$$x^3 = 64$$

$$x = 4$$

- An equation of the form $a^x = b$ can be solved by taking logs of both sides

## 9 Differentiation and Integration

- If $y = x^n$ then $\frac{dy}{dx} = nx^{n-1}$
- $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$ for all values of n except n = – 1

e.g $\int \frac{1 + x^2}{\sqrt{x}} = \int x^{-\frac{1}{2}} + x^{\frac{3}{2}} = 2x^{\frac{1}{2}} + \frac{2}{5}x^{\frac{5}{2}} + c$

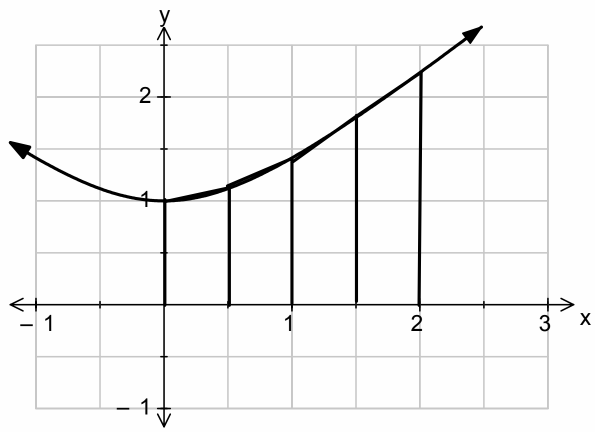
<u>TRAPEZIUM RULE</u>

The trapezium rule gives an approximation of the area under a graph.
If the gap between the ordinates is h then

Area = ½ h (end ordinates + twice 'sum of interior ordinates')

Use the trapezium rule 4 strips to estimate the area under the graph of $y = \sqrt{1 + x^2}$ from $x = 0$ to $x = 2$

| x | y |
|---|---|
| 0 | 1 |
| 0.5 | √1.25 |
| 1 | √2 |
| 1.5 | √3.25 |




Area =
½ x 0.5 ( 1 + √3.25 + 2(√1.25 + √2))
= 1.966917
= 1.97 (3sf)