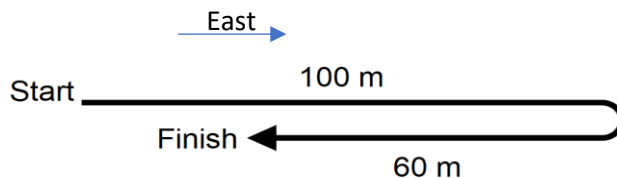


AS MATHS - MECHANICS REVISION NOTES

1 KINEMATICS

- **Distance** - a scalar quantity with no direction
= 160 m
- **Displacement** - a vector quantity – measured from the starting position
= 40 m (East of starting point)
- **Position** - a vector quantity – distance from a fixed origin

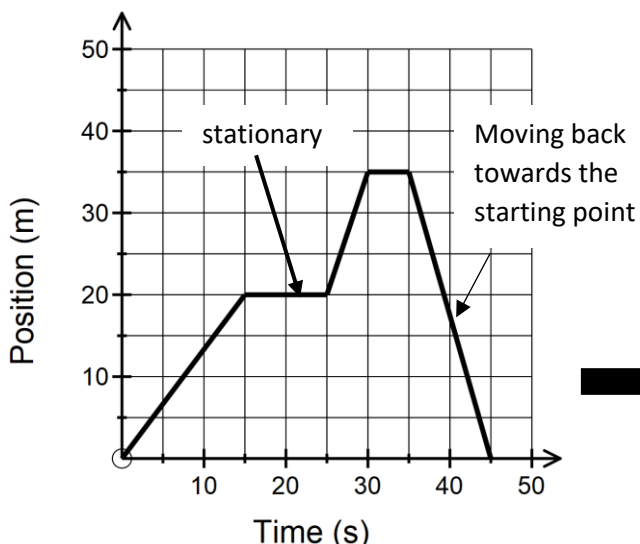


$$\text{AVERAGE SPEED} = \frac{\text{Total Distance}}{\text{Total Time}}$$

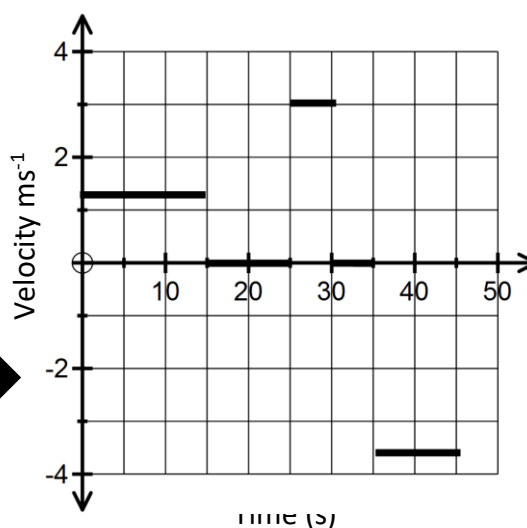
$$\text{AVERAGE VELOCITY} = \frac{\text{Displacement}}{\text{Time taken}}$$

USING GRAPHS

Position- time graph

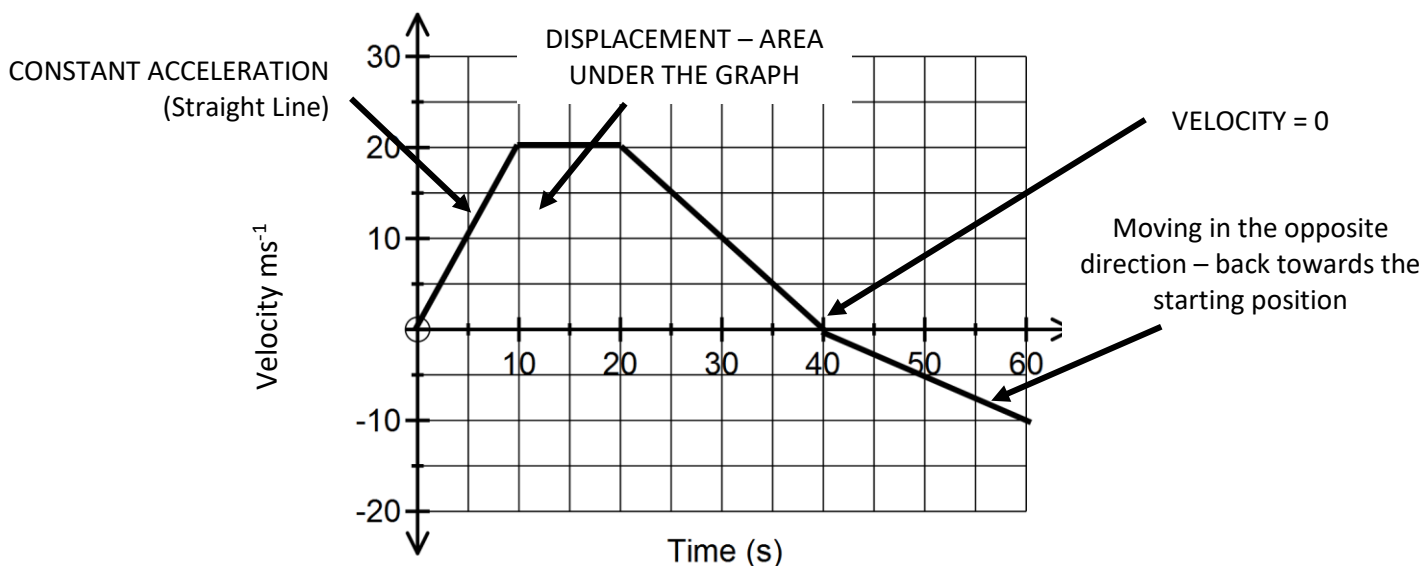


Velocity – time graph



VELOCITY TIME GRAPH

Gradient = acceleration



EQUATIONS FOR CONSTANT ACCELERATION

s : displacement (m) u : initial velocity (ms^{-1}) v : final velocity (ms^{-1}) a : acceleration (ms^{-2})

t = time (s)

$$v = u + at$$

$$v^2 = u^2 + 2as$$

$$s = \frac{1}{2}(u + v)t$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

- Acceleration due to gravity is **9.8 ms^{-2}** (unless given in the question)
- Negative Acceleration – retardation/deceleration

A car starts from rest and reaches a speed of 15 ms^{-1} after travelling 25m with constant acceleration. Assuming the acceleration remains constant, how much further will the car travel the next 4 seconds?

$$\begin{aligned}u &= 0 \text{ ms}^{-1} \\v &= 15 \text{ ms}^{-1} \\s &= 25 \text{ m}\end{aligned}$$

$$v^2 = u^2 + 2as \quad 15^2 = 2a \times 25 \\a = 4.5 \text{ ms}^{-2}$$

$$\begin{aligned}u &= 15 \text{ ms}^{-1} \\t &= 4 \\a &= 4.5\end{aligned}$$

$$\begin{aligned}s &= ut + \frac{1}{2}at^2 \\s &= 15 \times 4 + \frac{1}{2} \times 4.5 \times 16 \\&= 96 \text{ m}\end{aligned}$$

A ball is thrown vertically upwards with a speed of 12 ms^{-1} from a height of 1.5 m. Calculate the maximum height reached by the ball.

$$\begin{aligned}u &= 12 \text{ ms}^{-1} \\a &= -9.8 \text{ ms}^{-2}\end{aligned}$$

At maximum height $v = 0$

$$\begin{aligned}v^2 &= u^2 + 2as \\0 &= 144 - 2 \times 9.8 \times s \\s &= 7.35 \text{ m}\end{aligned}$$

$$\begin{aligned}\text{Maximum height} &= 1.5 + 7.35 \\&= 8.85 \text{ m}\end{aligned}$$

2 FORCES and ASSUMPTIONS

KEY FORCES

W : weight ($mg = \text{mass} \times 9.8$)

R : reaction (normal reaction – at right angles to the point of contact)

F : friction (acts in a direction opposite to that in which the object is moving or is on the point of moving)

T : Tension

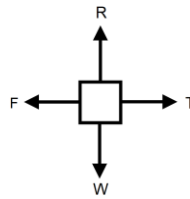
ASSUMPTIONS

- Motion is in a straight line
- Air Resistance can be ignored
- Objects are modelled as masses concentrated at a single point – no rotation
- Strings and rods are inextensible (no stretch) and are 'light' – mass can be disregarded
- Pulleys are smooth – no friction

3 NEWTONS LAWS

1st LAW : Every object remains at rest or moves with constant velocity unless an external force is applied

The system is in **EQUILIBRIUM**



$$T = F$$

$$R = W$$

2nd LAW : A force acting on an object is equal to the acceleration of that body times its mass.

$$F = ma$$

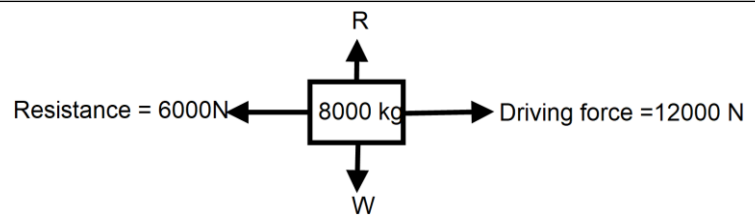
3rd LAW : If an object A exerts a force on object B, then object B must exert a force of equal magnitude and opposite direction back on object A.

Calculate the acceleration of the object

$$\text{Resultant force} = 12000 - 6000$$

$$= 6000 \text{ N}$$

$$6000 = 8000a \quad a = 0.75 \text{ ms}^{-2}$$



A man of mass 80 kg stands in a lift

Calculate the normal reaction of the lift floor on the man if

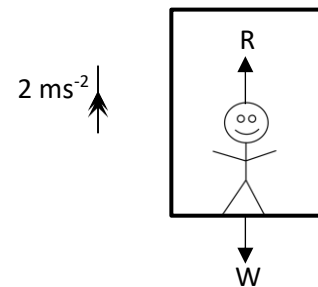
- a) The lift is moving downwards with constant velocity
Constant velocity so $R = W$

$$R = 80g \text{ N}$$

$$= 784 \text{ N}$$

- b) The lift is moving upwards with acceleration of 2 ms^{-2}
Upwards movement so $R > W$ $R - 80g = 80 \times 2$

$$R = 944 \text{ N}$$



Two masses are connected by a light string passing over a smooth pulley as shown below.

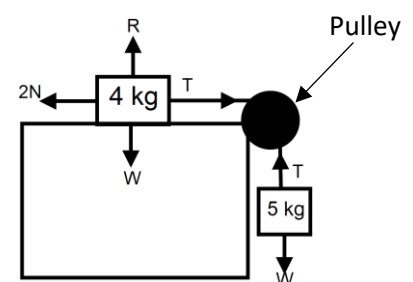
Calculate the acceleration of the 4 kg block when released from rest.

$$5 \text{ kg Block} : 5g - T = 5a$$

$$4 \text{ kg Block} : T - 2 = 4a$$

$$\text{Solving simultaneously} \quad 5g - 2 = 9a$$

$$a = 5.22 \text{ ms}^{-2}$$



Forces $F_1 = 2i + j$, $F_2 = -3i + 4j$ and $F_3 = 4i - 6j$ act on a particle with mass 10 kg. Find the magnitude of acceleration of the particle

$$\text{Resultant force} = F_1 + F_2 + F_3 = (2i + j) + (-3i + 4j) + (4i - 6j)$$

$$= 3i - j$$

$$F = ma$$

$$3i - j = 10a \quad a = 0.3i - 0.1j \quad |a| = \sqrt{0.3^2 + (-0.1)^2} \quad a = 0.316 \text{ ms}^{-2}$$

4 VARIABLE ACCELERATION (r : position)

Position r (m) Velocity v (ms^{-1}) Acceleration a (ms^{-2})



$$r = f(t)$$

$$v = \frac{dr}{dt}$$

$$a = \frac{dv}{dt} \quad \left(\frac{d^2r}{dt^2} \right)$$



$$r = \int v \, dt$$

$$v = \int a \, dt$$

$$a = f(t)$$

Remember

- Area under a velocity time graph = displacement
- Gradient at a point on position/time graph = velocity
- Gradient at a point on velocity/time graph = acceleration

The acceleration of a particle (in ms^{-2}) at time t seconds is given by $a = 12 - 2t$.
The particle has an initial velocity of 3 ms^{-1} when it starts at the origin.

a) Find the velocity of the particle after t seconds

$$v = \int 12 - 2t \, dt$$

$$v = 12t - t^2 + c \quad t = 0 \quad v = 3 \quad c = 3$$

$$v = 12t - t^2 + 3$$

b) Find the position of the particle after t seconds

$$r = \int 12t - t^2 + 3 \, dt$$

$$= 6t^2 - \frac{t^3}{3} + 3t + c$$

$$r = 0 \quad t = 0 \quad r = 6t^2 - \frac{t^3}{3} + 3t$$

A train moves between 2 stations, stopping at both of them

It's speed at t seconds is modelled by $V = \frac{1}{5000} t(1200 - t)$ (ms^{-1})

Find the distance between the 2 stations

$$\text{At the stations } v = 0 \quad \frac{1}{5000} t(1200 - t) = 0 \quad t = 0 \quad t = 1200$$

$$\text{Distance} = \int_0^{1200} \frac{1}{5000} t(1200 - t) dt = \frac{1}{5000} \left[600t^2 - \frac{t^3}{3} + c \right]$$

$$= 57600 \text{ m}$$

$$= 57.6 \text{ km}$$