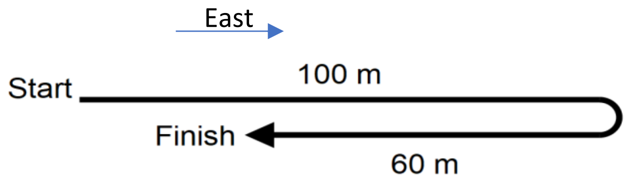
AS MATHS - MECHANICS REVISION NOTES

1 KINEMATICS

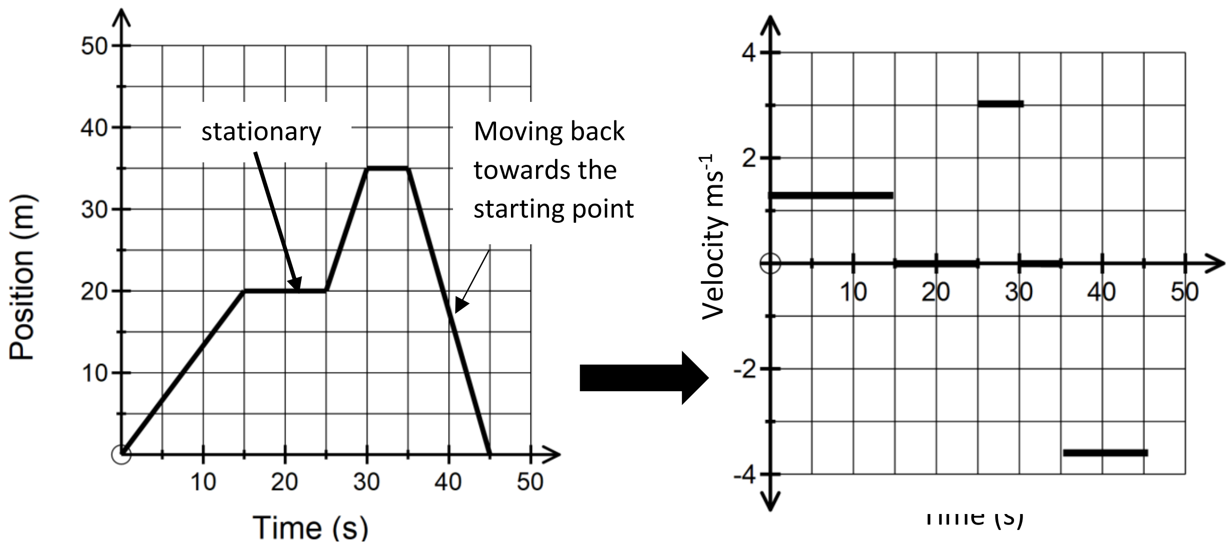


- **Distance** - a scalar quantity with no direction
  = 160 m
- **Displacement** - a vector quantity – measured from the starting position
  = 40 m (East of starting point)
- **Position** - a vector quantity – distance from a fixed origin

AVERAGE SPEED = $\frac{Total\ Distance}{Total\ Time}$ **AVERAGE VELOCITY**= $\frac{Displacement}{Time\ taken}$

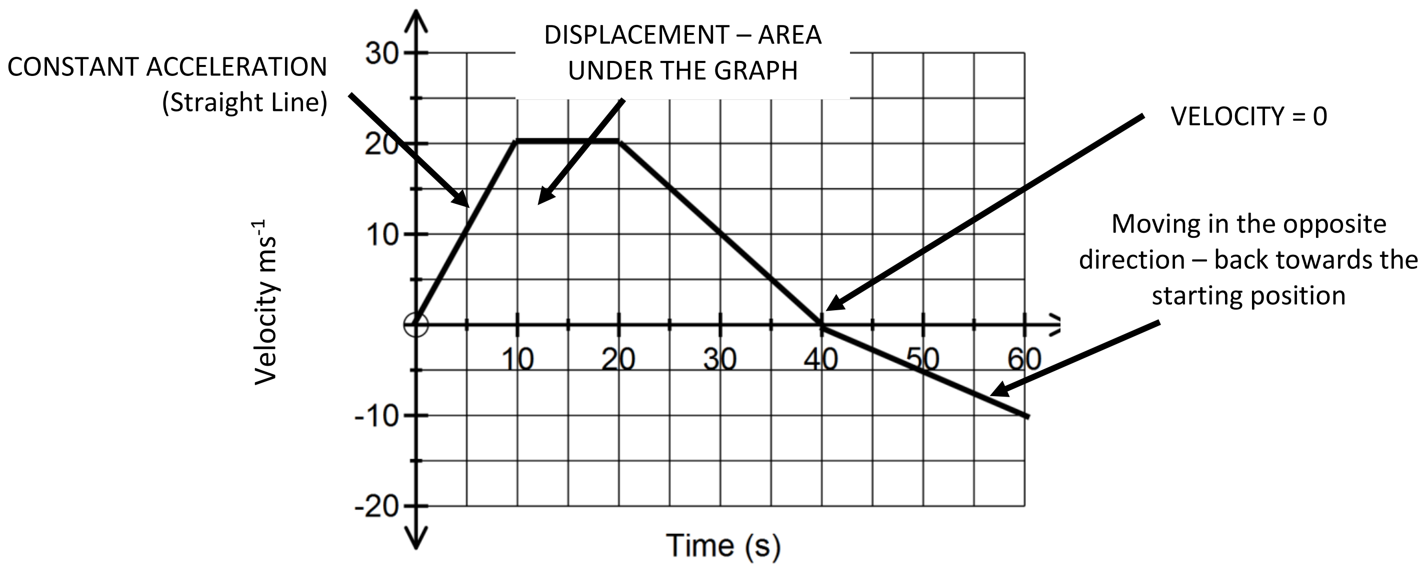
USING GRAPHS

Position- time graph **Velocity – time graph**



VELOCITY TIME GRAPH

Gradient = acceleration



EQUATIONS FOR CONSTANT ACCELERATION

s: displacement (m) u : initial velocity (ms^{-1}) v : final velocity (ms^{-1}) a : acceleration (ms^{-2})

t = time (s)

$$v = u + at \qquad v^2 = u^2 + 2as$$

$$s = \tfrac{1}{2}(u + v)t \qquad s = ut + \tfrac{1}{2}at^2 \qquad s = vt - \tfrac{1}{2}at^2$$

- Acceleration due to gravity is **9.8 ms^{-2}** (unless given in the question)
- Negative Acceleration – retardation/deceleration

A car starts from rest and reaches a speed of 15 ms^{-1} after travelling 25m with constant acceleration. Assuming the acceleration remains constant, how much further will the car travel the next 4 seconds?

u = 0 ms^{-1}
v = 15 ms^{-1}
s = 25 m

$v^2 = u^2 + 2as \quad 15^2 = 2a \times 25$
$a = 4.5\ ms^{-2}$

u = 15 ms^{-1}
t = 4
a = 4.5 $s = ut + \tfrac{1}{2}at^2$
$s = 15 \times 4 + \tfrac{1}{2} \times 4.5 \times 16$
$= 96$ m

A ball is thrown vertically upwards with a speed of 12 ms^{-1} from a height of 1.5 m. Calculate the maximum height reached by the ball.

u = 12 ms^{-1}
a = -9.8 ms^{-2}
At maximum height v = 0
$v^2 = u^2 + 2as$
$0 = 144 - 2\times9.8\times s$
$s = 7.35$ m

Maximum height = 1.5 + 7.35
= 8.85 m

2 FORCES and ASSUMPTIONS

KEY FORCES

W : weight (mg = mass × 9.8)
R : reaction (normal reaction – at right angles to the point of contact)
F : friction (acts in a direction opposite to that in which the object is moving or is on the point of moving)
T : Tension

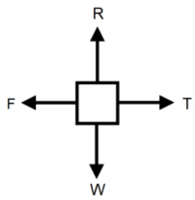
ASSUMPTIONS

- Motion is in a straight line
- Air Resistance can be ignored
- Objects are modelled as masses concentrated at a single point – no rotation
- Strings and rods are inextensible (no stretch) and are 'light' – mass can be disregarded
- Pulleys are smooth – no friction

3 NEWTONS LAWS

1st LAW : Every object remains at rest or moves with contact velocity unless an external force is applied

The system is in **EQUILIBRIUM**



T = F
R = W

2nd LAW : A force acting on an object is equal to the acceleration of that body times its mass.

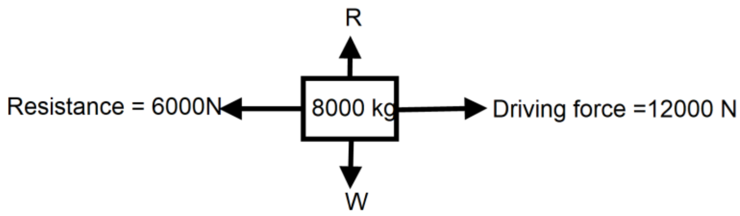
F = ma

3rd LAW : If an object A exerts a force on object B, then object B must exert a force of equal magnitude and opposite direction back on object A.

Calculate the acceleration of the object

Resultant force = 12000 – 6000
= 6000 N

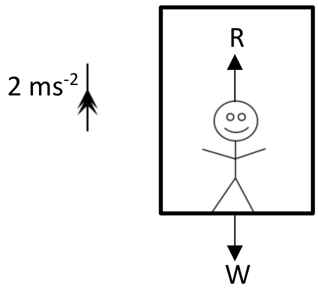
6000 = 8000a   a = 0.75 ms^{-2}



A man of mass 80 kg stands in a lift
Calculate the normal reaction of the lift floor on the man if

a) The lift is moving downwards with constant velocity
Constant velocity so R = W
R = 80g N
= 784 N

b) The lift is moving upwards with acceleration of 2 ms^{-2}
Upwards movement so R > W   R – 80g = 80×2
R = 944 N

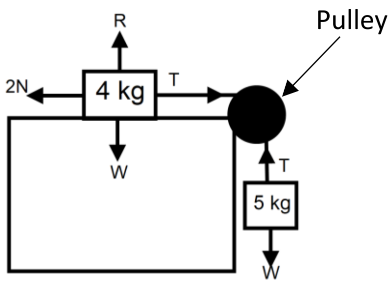


Two masses are connected by a light string passing over a smooth pulley as shown below.
Calculate the acceleration of the 4 kg block when released from rest.

5 kg Block : 5g – T = 5a
4kg Block : T – 2 = 4a

Solving simultaneously 5g – 2 = 9a

a = 5.22 ms^{-2}



Forces $F_1 = 2i + j$, $F_2 = -3i + 4j$ and $F_3 = 4i – 6j$ act on a particle with mass 10 kg. Find the magnitude of acceleration of the particle

Resultant force = $F_1 + F_2 + F_3 = (2i + j) + (-3i + 4j) + (4i – 6j)$
$= 3i – j$

F = ma

$3i – j = 10a$ $a = 0.3i - 0.1j$ $|a| = \sqrt{0.3^2 + (-0.1)^2}$ a = 0.316 ms^{-2}

4 VARIABLE ACCELERATION (r : position)

Position r (m) **Velocity v (ms^{-1})** **Acceleration a (ms^{-2})**

DIFFERENTIATING →

r = f(t) v = $\frac{dr}{dt}$ a = $\frac{dv}{dt}$ $\left(\frac{d^2r}{dt^2}\right)$

← **INTEGRATING - Don't forget to include + c**

r = $\int v \; dt$ v = $\int a \; dt$ a = f(t)

Remember

- Area under a velocity time graph = displacement
- Gradient at a point on position/time graph = velocity
- Gradient at a point on velocity/time graph = acceleration

The acceleration of a particle (in ms^{-2}) at time t seconds is given by a = 12 – 2t.
The particle has an initial velocity of 3 ms^{-1} when it starts at the origin.

a) Find the velocity of the particle after t seconds

v = $\int 12 - 2t \; dt$

v = 12t – t^2 + c t = 0 v = 3 c = 3

v = 12t – t^2 + 3

b) Find the position of the particle after t seconds

r = $\int 12t - t^2 + 3 \; dt$

= $6t^2 - \frac{t^3}{3} + 3t + c$

r = 0 t = 0 r = $6t^2 - \frac{t^3}{3} + 3t$

A train moves between 2 stations, stopping at both of them
It's speed at t seconds is modelled by V = $\frac{1}{5000}$t(1200 – t) (ms^{-1})
Find the distance between the 2 stations

At the stations v = 0 $\frac{1}{5000}$t(1200 – t) = 0 t = 0 t = 1200

Distance = $\int_0^{1200} \frac{1}{5000} t(1200 - t) dt = \frac{1}{5000}\left[600t^2 - \frac{t^3}{3} + c\right]$

= 57600 m
= 57.6 km