MathsBox

AS & A Level

QUESTION GENERATOR OVERVIEW

Question structures for A Level Further Maths.

Every question structure available for A Level Further Maths in the Mathsbox Question Generator, organised by topic so you can find exactly what you need.

- Complex Numbers
- Matrices
- Further Algebra
- Further Calculus
- Further Vectors
- Hyperbolic Functions
- Polar Coordinates
- Sketching Graphs
- Differential Equations
- Numerical Integration
- Mechanics
- Discrete Mathematics
- Statistics

1,060
question structures

102
topics

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A LEVEL FURTHER MATHS

01 Complex Numbers

6 topics
78 structures

Arithmetic

AS 9

Addition
Subtraction
Multiplication
Linear combinations
Division
Powers of i
Real and imaginary parts
Adding three complex numbers
Complex Numbers Arithmetic : Mixture

Conjugates and equations

AS 9

Write down the conjugate
Find $z\bar{z}$
Find $z + \bar{z}$ or $z - \bar{z}$
Solve a quadratic with complex roots
Find the quadratic given one root
Cubic with one real root given
Find an unknown coefficient
Solve an equation involving z and $\bar{z}$
Conjugates and equations : Mixture

Modulus-argument form

AS 11

Find the modulus
Find the argument
Write in modulus-argument form
Modulus-argument to Cartesian form
Multiplying in modulus-argument form
Dividing in modulus-argument form
Modulus of a product
Argument of a product
Distance from the origin - Argand diagram
Find z given modulus and argument
Modulus-argument form : Mixture

Loci

AS 17

Circle: state centre and radius
Circle: state centre and radius (expanded form)
Circle- equation
Circle - equation (expanded form)
Perpendicular bisector: Cartesian equation
Half-line: find a point on the locus
Half-line: find a point (exact angles)
Circle meeting the real axis
Circle: minimum $|z|$

Loci *continued*

Circle: maximum $|z|$
Perpendicular bisector: verify the midpoint
Circle meeting a horizontal line
Circle: minimum distance to another point
Circle meeting the imaginary axis
Perpendicular bisector - real axis
Perpendicular bisector - real axis -locus
Loci : Mixture

De Moivre's theorem

A LEVEL 18

Cartesian to exponential form
Modulus-argument to exponential form
Exponential form to Cartesian
Modulus of a product (exponential form)
Modulus of a quotient (exponential form)
Argument of a product (exponential form)
Argument of a quotient (exponential form)
Express $(\cos\theta + i\sin\theta)^n$ in Cartesian form
Power of z in polar form
Power of z in Cartesian form
Express $a\cos(n\theta)$ or $a\sin(n\theta)$ in powers
Powers of sin/cos in multiple-angle form
Multiply powers using De Moivre
Modulus and argument of a power
Negative power of z
Re and Im of a power (decimal angle)
Smallest n for real z^n or z^n=1
De Moivre's theorem : Mixture

Roots of complex numbers

A LEVEL 14

nth roots of unity (polar form)
Root with smallest positive argument
nth roots of unity (Cartesian form)
Solve $z^n = \text{positive real}$ (Cartesian)
Solve $z^n = \text{negative real}$ (Cartesian)
Solve $z^n = ai$ (exponential form)
Solve $z^n + c = 0$ (exponential form)
Solve $az^n + c = 0$ (exponential form)
Solve $z^n \pm ci = 0$ (exponential form)
Solve $az^n \pm bi = c$ (exponential form)
Evaluate expressions involving roots of unity
Area/perimeter of roots on Argand diagram
Area/perimeter for $z^n = a + bi$
Roots of complex numbers : Mixture

02 A LEVEL FURTHER MATHS Matrices

5 topics
64 structures

Operations

AS 9

- Matrix addition
- Matrix subtraction
- Scalar multiplication
- Matrix multiplication (2×2)
- Matrix times a vector
- Linear combination of matrices
- Solve a matrix equation $A + X = B$
- Multiplying by the identity matrix
- Matrices Operations : Mixture

Determinants and inverses

AS 12

- Find $(AB)^T$ using transposed matrices
- Find the determinant
- Find k for a singular matrix
- Find the inverse matrix
- Determine whether a matrix is singular
- Simultaneous equations - inverse matrix
- $\det(AB)$ given $\det(A)$ and $\det(B)$
- Find X given $XA = I$
- $\det(kA)$ for a 2×2 matrix
- Solve a matrix equation $AX = B$
- Find A given A^{-1}
- Determinants and inverses : Mixture

Linear transformations

AS 14

- State a rotation matrix
- State a reflection matrix
- State an enlargement matrix
- Matrix from a described transformation (mixed)
- Find the image of a point under a matrix
- Find the image of a point under a rotation
- Combined transformation matrix BA
- Combined transformation matrix AB
- Describe a transformation given its matrix
- Apply an enlargement to a vector
- Combined named transformations - matrix

Linear transformations *continued*

- Combined named transformations - point
- State the scale factor of an enlargement
- Linear transformations : Mixture

Invariant points and lines

AS 8

- Find the line of invariant points
- Find invariant line(s) through the origin
- State the invariant line for a reflection
- Repeated invariant line through the origin
- Find an unknown given an invariant line
- Show a line is invariant - scale factor
- Show there are no invariant lines
- Invariant points and lines : Mixture

3x3 matrices

A LEVEL 21

- Determinant of a 3x3 matrix
- Volume of image under a matrix transformation
- Determine if a 3x3 matrix is singular
- Inverse of a 3x3 matrix
- Inverse of a 3x3 matrix in terms of k
- Find A given $AM = P$
- Find B given $MB = P$
- Find BA given A and AB
- Solve 3 equations using the inverse matrix
- Find a cofactor
- Find a minor
- Find k for a singular matrix
- det(kA) for a 3x3 matrix
- Related determinant - row/column change
- Eigenvalues of a 2x2 matrix
- Eigenvalues of a 3x3 matrix
- Eigenvectors of a 2x2 matrix
- Eigenvectors of a 3x3 matrix
- Diagonalise a 2x2 matrix
- Diagonalise a 3x3 matrix
- 3x3 matrices : Mixture

03 A LEVEL FURTHER MATHS Further Algebra

4 topics
55 structures

Proof

AS 11

- Verify the base case for a sum formula
- Write down the inductive hypothesis
- Verify the inductive step numerically
- Verify a divisibility base case
- Verify a divisibility statement for $n = 2$
- Matrix power conjecture for a given n
- Diagonal matrix power conjecture - given n
- Verify the sum of squares formula
- Complete a divisibility inductive step
- Full proofs by induction (various)
- Proof : Mixture

Series

AS 17

- Evaluate a sum - non-standard limits
- Evaluate a linear sum using standard results
- Evaluate a linear sum - non-standard limits
- Evaluate $\sum r^2$ - standard result
- Evaluate $\sum r^2$ - non-standard limits
- Evaluate $\sum r^3$ - standard result
- Evaluate $\sum r^3$ - non-standard limits
- Quadratic sum using standard results
- Quadratic sum - non-standard limits
- Sum of an arithmetic series
- Sum of a geometric series

Series *continued*

- Sum to infinity of a geometric series
- Method of differences (telescoping)
- Evaluate a telescoping sum (various forms)
- Evaluate a telescoping sum in terms of n
- Method of differences in terms of n
- Series : Mixture

Roots of polynomials — AS — 18

- Sum and product of roots (quadratic)
- Find $\alpha^2 + \beta^2$
- Find $\frac{1}{\alpha} + \frac{1}{\beta}$
- Find a new equation with scaled roots
- Sums and products of roots (cubic)
- Find $\alpha^2 + \beta^2 + \gamma^2$ (cubic)
- Find $\alpha^3 + \beta^3 + \gamma^3$ (cubic)
- Find $(1+\alpha)(1+\beta)(1+\gamma)$ (cubic)
- Find $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ (cubic)
- Find $(\alpha\beta)^2 + (\alpha\gamma)^2 + (\beta\gamma)^2$ (cubic)
- New cubic with roots $\alpha \pm k$

Roots of polynomials *continued*

- New cubic with reciprocal roots
- New cubic with scaled roots α/m
- New cubic with roots $\alpha/m \pm k$
- Find the real root given a complex root
- Cubic equation - three given roots
- Find the quadratic factor given one root
- Roots of polynomials : Mixture

Solving Inequalities — AS — 9

- Quadratic inequalities
- Cubic inequalities - factorised form
- Cubic inequalities
- Quartic inequalities
- Rational: linear over linear
- Rational: product over linear
- Rational: linear over product
- Rational: cubic over linear
- Solving Inequalities : Mixture

04 A LEVEL FURTHER MATHS — Further Calculus

11 topics
114 structures

Volumes and mean values · Volume — AS — 14

- Volume of a cone from $y = mx$
- Volume of revolution about the x-axis
- Volume of revolution over a non-zero interval
- Volume of revolution: $y = k/x$
- Region $y = a - bx^3$ about the x-axis
- Region $y = ax^4 - bx^3$ about the x-axis
- Region $y = a - bx^3$ about the y-axis
- Shaded region: parabola, line and x-axis
- Region R: $y = \sqrt{kx}$, line and x-axis
- Shaded region: parabola, $y = k$ and x-axis
- Volume of revolution about the y-axis
- Volume of revolution: $y^2 = kx$
- Region R about the y-axis
- Volumes : Mixture (volumes)

Volumes and mean values · Mean Value — AS — 6

- Mean value of kx^n
- Mean value of a linear function
- Mean value of a quadratic function
- Mean value of a cubic function
- Find the integral given the mean value
- Mean Value : Mixture

Volumes and mean values · Mixture — AS — 1

- Volumes and mean values : Mixture

Maclaurin series — AS — 9

- Maclaurin series for ae^{bx}
- Maclaurin series for $a\sin(bx)$
- Maclaurin series for $a\cos(bx)$
- Maclaurin series for $a\ln(1+bx)$
- Maclaurin series for $\frac{1}{ax+b}$
- Maclaurin series for $\sqrt{1+ax}$

Maclaurin series *continued*

- Maclaurin series for $(1+ax)^{b/c}$
- Maclaurin series for $\sin(x^n)$ or $\cos(x^n)$
- Maclaurin series : Mixture

Maclaurin series — A LEVEL — 11

- Expansion of $(1+x)^p$
- Expansion of $(1+ax)^p$
- Expansion of $(a \pm x)^n$
- Product of series - First 4 non-zero terms
- Limits as $n \to \infty$ of rational functions
- Limits using L'Hôpital's rule
- Simple limits as $x \to 0$ (0/0 form)
- Trickier limits as $x \to 0$ (higher-order 0/0)
- Harder limits as $x \to 0$ (composites)
- Approximating - $\ln(1+x)$
- Maclaurin series : Mixture

Improper integrals — A LEVEL — 11

- Infinite upper limit: exponential
- Infinite limit: power x^{-n}
- Infinite upper limit- converge/diverge
- Values of p for convergence
- Unbounded integrand near 0: evaluate
- Unbounded integrand - converge/diverge
- Infinite lower limit: exponential
- Infinite upper limit: $(ax+b)^{-2}$
- Evaluate or show divergence (mixed type)
- Use a limit to evaluate an improper integral
- Improper integrals : Mixture

Integrating inverse trig functions

A LEVEL 21

- Integrate $\frac{1}{\sqrt{a^2-x^2}}$ (arcsin)
- Integrate $\frac{1}{(a^2+x^2)}$ (arctan)
- Integrate $\frac{k}{\sqrt{a^2-x^2}}$
- Integrate $\frac{k}{a^2+x^2}$
- Integrate $\frac{1}{\sqrt{a^2-b^2x^2}}$
- Integrate $\frac{1}{a^2+b^2x^2}$
- Complete the square then arctan
- Complete the square then arcsin
- Definite integral (arcsin form)
- Definite integral (arctan form)
- Definite integral $\frac{k}{\sqrt{a^2-x^2}}$
- Definite integral $\frac{k}{a^2+x^2}$
- Definite integral $\frac{1}{\sqrt{a^2-b^2x^2}}$
- Definite integral $\frac{1}{a^2+b^2x^2}$
- Integrate linear-over-quadratic (arctan)
- Split linear over $\sqrt{a^2-x^2}$
- Split linear over $a^2+b^2x^2$
- Split $\frac{x^2+k}{x^2+a^2}$
- Split quadratic over x^2+a^2
- Split linear over $\sqrt{a^2-b^2x^2}$
- Inverse trig functions : Mixture

Integrating with partial fractions

A LEVEL 13

- Numerator constant, two linear factors
- Integrate after stating partial fractions
- Two distinct linear factors
- Difference of squares denominator
- Repeated linear factor
- Three distinct linear factors
- Linear and irreducible quadratic
- Improper rational: divide then integrate
- Repeated factor and a distinct linear factor
- Definite integral with two linear factors
- Definite integral with a repeated factor
- Definite integral mixture
- Partial fractions - integrals : Mixture

Reduction formulae

A LEVEL 15

- Apply a given reduction formula
- Reduction formula for $\int \sin^n x\,dx$
- Reduction formula for $\int \cos^n x\,dx$
- Reduction formula for $\int \tan^n x\,dx$
- Definite reduction: $\int_0^{\pi/2} \sin^n x\,dx$
- Evaluate $\int_0^{\pi/2} \sin^n x\,dx$ (even n)
- Evaluate $\int_0^{\pi/2} \sin^n x\,dx$ (odd n)
- Evaluate $\int_0^{\pi/2} \cos^n x\,dx$
- Reduction for $\int_0^{\pi/4} \tan^n x\,dx$
- Reduction formula for $\int x^n e^{ax}\,dx$
- Use reduction to integrate $x^n e^{ax}$
- Definite: $x^n \sin x$ and $x^n \cos x$
- Definite: $(\ln x)^n$ and $x^m(\ln x)^n$
- Definite mixture: $x^n \sin x$, $x^n \cos x$, $(\ln x)^n$
- Reduction formulae : Mixture

Length and surface area

A LEVEL 10

- Arc length of $y = f(x)$
- Arc length: hyperbolic and trig curves
- Arc length: parametric curve
- Arc length of a circular arc
- Arc length using $x = g(y)$
- Set up an arc length or surface integral
- Surface area - about x-axis
- Surface area - about y-axis
- Surface area of revolution
- Length and surface area : Mixture

Trapezium rule

A LEVEL 3

- Find the strip width h
- Trapezium rule for $y = f(x)$
- Trapezium rule : Mixture

05 A LEVEL FURTHER MATHS Further Vectors

2 topics
44 structures

Lines in 3D

AS 20

- Vector equation given a point and direction
- Vector equation through two points
- Cartesian equation through two points
- Cartesian to vector form
- Image of a line under a 2×2 matrix
- Image of a line under a 3×3 matrix
- Parametric equations of a line
- Determine whether a point lies on a line
- Scalar (dot) product of two vectors
- Angle between two vectors
- Find a for perpendicular vectors
- Acute angle between two lines
- Show two lines intersect and find the point
- Point and obtuse angle between lines
- Parallel, intersecting (point) or skew ?
- Distance between two points

Lines in 3D *continued*

- Minimum distance from a point to a line
- Reflection of a point in a line
- Find a unit vector
- Vectors - Lines in 3D : Mixture

Planes and lines

A LEVEL 24

- Vector product of two 3D vectors
- Vector perpendicular to two 3D vectors
- Vector product - angle
- Area of a triangle from three vertices
- Line in the form $(\mathbf{r}-\mathbf{a})\times\mathbf{b}=\mathbf{0}$
- Line through two points - $\mathbf{r}\times\mathbf{b}=\mathbf{a}$
- Cartesian to $(\mathbf{r}-\mathbf{a})\times\mathbf{b}=\mathbf{0}$
- Vector equation of a plane
- Cartesian to vector form
- Plane through three points (vector form)

Planes and lines *continued*

- Plane through three points (scalar product form)
- Plane through three points (Cartesian form)
- Angle between a line and a plane
- Angle between two planes
- Obtuse angle between two planes
- Acute angle - a plane and a line (Cartesian)
- Obtuse angle - a plane and a line (Cartesian)

Planes and lines *continued*

- Determine if a line is parallel to a plane
- Find parameter so a point lies on a plane
- Plane through a point perpendicular to a line
- Intersection of a line and a plane
- Intersection of a line/plane (vector forms)
- Distance from a point to a plane
- Planes and lines : Mixture

06 A LEVEL FURTHER MATHS Hyperbolic Functions

3 topics
45 structures

Definitions and identities — AS — 10

- Evaluate $\sinh x$
- Evaluate $\cosh x$
- Evaluate $\tanh x$
- Find $\sinh/\cosh$ from identity
- Solve hyperbolic equations
- Find $\operatorname{sech}^2 x$ from $\tanh x$
- Solve $\cosh x = k$
- Solve $\sinh x = k$
- Solve $\tanh x = p/q$
- Hyperbolic functions : Mixture

Identities and double-angle formulae — A LEVEL — 12

- Find $\sinh(2x)$ or $\cosh(2x)$
- Find $\tanh(2x)$ from $\tanh x$
- Addition formulae
- Solve simple equations $f(2x) = k$
- Solve equations - double-angle identities
- Solve more complex equations
- Solve $\sinh x = a \operatorname{sech} x$
- Solve $\cosh x = a \operatorname{cosech} x$
- Solve $\coth x = a - \tanh x$
- Solve $\operatorname{cosech}^2 x - \tanh x = a$
- Solve $\operatorname{sech}^2 x - \tanh^2 x = a - b \operatorname{sech} x$
- Hyperbolic Identities & double-angle : Mixture

Calculus of hyperbolics — A LEVEL — 23

- Differentiate $\mathrm{k}\sinh ax$
- Differentiate $\mathrm{k}\cosh ax$
- Differentiate $\mathrm{k}\tanh ax$
- Differentiate $\sinh$ expressions
- Differentiate $\cosh$ expressions
- Differentiate $\tanh$ expressions
- Differentiate $\coth$ expressions
- Differentiate sech expressions
- Differentiate cosech expressions
- Differentiate hyperbolic expressions
- Differentiate $\mathrm{f}(\mathrm{x})=\mathrm{a}\sinh x + b\cosh x$
- Differentiate and solve
- Integrate $\mathrm{k}\cosh(ax)$
- Integrate $\mathrm{k}\sinh(ax)$
- Integrate $\mathrm{k}\tanh(ax)$
- Integrate hyperbolic expressions
- Integral formula with arsinh
- Integral formula with $\arccosh$
- Definite integral of $\cosh$
- Definite integral of $\sinh$
- Definite integrals of hyperbolic expressions
- Definite integrals with $\operatorname{arsinh}/\arccosh$
- Hyperbolics -calculus : Mixture

07 A LEVEL FURTHER MATHS Polar Coordinates

3 topics
23 structures

Curves and points — AS — 9

- Polar to Cartesian
- Cartesian to polar
- Evaluate r for a polar curve
- Convert a polar line to Cartesian
- Sketch polar curve: max/min r
- Convert a circle to polar form
- Convert a polar equation to Cartesian
- Points of intersection of polar curves
- Polar coordinates : Mixture

Sketching graphs — AS — 6

- Sketching $r = a(1 \pm \cos\theta)$
- Sketching $r = a(1 \pm \sin\theta)$
- Sketching a limaçon
- Sketching a rose curve
- Sketching a circle through the origin
- Polar coordinates - Sketching : Mixture

Areas — A LEVEL — 8

- Area of a sector (constant r)
- $\mathrm{r}=\mathrm{a}\sin(b\theta)$ or $\mathrm{r}=\mathrm{a}\cos(b\theta)$
- $\mathrm{r}=\mathrm{a}\sin(b\theta)$ or $\mathrm{r}=\mathrm{a}\cos(b\theta)$ $r \geq 0$
- $\mathrm{r}=\mathrm{a}(1+\cos\theta)$ or $\mathrm{r}=\mathrm{a}(1+\sin\theta)$

Areas

continued

- r=a+bcos θ or r=a+bsin θ
- Common to r=acos θ / r=bsin θ

Areas

continued

- Between two polar curves
- Area - polar coordinates : Mixture

08 A LEVEL FURTHER MATHS Sketching Graphs

11 topics
68 structures

Linear rational functions

AS 6

- Sketching $y = \frac{k}{x}$
- Sketching $y = \frac{k}{x \pm a}$
- Sketching $y = b + \frac{k}{x \pm a}$
- Sketching $y = \frac{ax \pm b}{x \pm c}$
- Sketching $y = \frac{ax \pm b}{cx \pm d}$
- Sketching graphs (1) : Mixture

Quadratic rational functions

AS 6

- Sketching $y = x + \frac{k}{x}$
- Sketching $y = x + a + \frac{k}{x \pm b}$
- Sketching $y = \frac{x^2 \pm px \pm q}{x \pm r}$
- Sketching $y = a + \frac{bx \pm c}{x^2 + d}$
- Sketching $y = \frac{ax^2 \pm bx \pm c}{x^2 \pm d}$
- Sketching graphs (2) : Mixture

Parabolas

AS 3

- Sketching $y = ax^2$ $(x^2 = 4ay)$
- Sketching $x = ay^2$ $(y^2 = 4ax)$
- Sketching graphs (3) : Mixture

Ellipses

AS 3

- Sketching an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$
- Sketching $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $b > a$
- Sketching graphs (4) : Mixture

Hyperbolas

AS 3

- Sketching a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$
- Sketching a hyperbola $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$
- Sketching graphs (5) : Mixture

Hyperbolic functions

AS 6

- Sketching $y = \sinh(ax)$
- Sketching $y = \cosh(ax)$
- Sketching $y = \tanh(ax)$
- Sketching $y = a\sinh(bx)$
- Sketching $y = a\cosh(bx)$
- Sketching graphs (6) : Mixture

Reciprocal and modulus graphs

A LEVEL 7

- Sketching $y = |ax \pm b|$
- Sketching $y = a|x| \pm b$
- Sketching $y = |ax^2 \pm bx \pm c|$
- Sketching a quadratic in $|x|$

Reciprocal and modulus graphs

continued

- Sketching $y = \frac{1}{ax \pm b}$
- Sketching $y = \frac{1}{ax^2 \pm bx \pm c}$
- Sketching graphs (7) : Mixture

Transforming conic graphs

A LEVEL 10

- Enlargement of a parabola
- Enlargement of an ellipse
- Enlargement of a hyperbola
- Rotation of a parabola
- Rotation of an ellipse
- Rotation of a hyperbola
- Translation of a parabola
- Translation of an ellipse
- Translation of a hyperbola
- Sketching graphs (8) : Mixture

Equations of transformed conics

A LEVEL 10

- Enlargement of a parabola
- Enlargement of an ellipse
- Enlargement of a hyperbola
- Rotation of a parabola
- Rotation of an ellipse
- Rotation of a hyperbola
- Translation of a parabola
- Translation of an ellipse
- Translation of a hyperbola
- Transformed conics : Mixture

Reciprocal and inverse hyperbolic graphs

A LEVEL 7

- Sketching $y = a\,\text{sech}(x \pm b)$
- Sketching $y = a\,\text{cosech}(x \pm b)$
- Sketching $y = a\coth(x \pm b)$
- Sketching $y = a\sinh^{-1}(x \pm b)$
- Sketching $y = a\cosh^{-1}(x \pm b)$
- Sketching $y = a\tanh^{-1}(x \pm b)$
- Sketching graphs (9) : Mixture

Improper rational functions

A LEVEL 7

- Sketching $y = \frac{\text{quadratic}}{\text{linear}}$
- Sketching $y = \frac{\text{cubic}}{\text{linear}}$
- Sketching $y = \frac{\text{cubic}}{\text{quadratic}}$
- Asymptotes of a quadratic over a linear
- Asymptotes of a cubic over a linear
- Asymptotes of a cubic over a quadratic
- Sketching graphs (10) : Mixture

A LEVEL FURTHER MATHS

09 Differential Equations

8 topics
56 structures

First order DEs

A LEVEL 14

- Separate variables (general solution)
- Separate variables (particular solution)
- Find the integrating factor
- Find the integrating factor (harder)
- Linear first order - general solution (1)
- Linear first order - general solution (2)
- Linear first order - particular solution (1)
- Linear first order - particular solution (2)
- Identify $P(x)$ and $Q(x)$
- General solution - integrating factor x^n
- Particular solution - integrating factor x^n
- Separate variables: $\frac{dy}{dx} = ky$
- Exponential growth/decay (numeric)
- First order DEs : Mixture

Second order DEs · Complementary function

A LEVEL 4

- Complementary function: distinct real roots
- Complementary function: repeated root
- Complementary function: complex roots
- CF - Second order DEs : Mixture

Second order DEs · Particular integrals

A LEVEL 6

- Particular integral: constant RHS
- Particular integral: linear RHS
- Particular integral: exponential RHS
- Particular integral: quadratic or cubic RHS
- Particular integral: trig RHS
- PI - Second order DEs : Mixture

Second order DEs · General solutions

A LEVEL 6

- General solution: constant RHS
- General solution: linear RHS
- General solution: exponential RHS
- General solution: quadratic or cubic RHS
- General solution: trig RHS
- GS - Second order DEs : Mixture

Second order DEs · Particular solutions

A LEVEL 6

- Particular solution: constant RHS + ICs
- Particular solution: linear RHS + ICs
- Particular solution: exponential RHS + ICs
- Particular solution: quadratic/cubic
- Particular solution: trig RHS + ICs
- PS - Second order DEs : Mixture

Second order DEs · Simple Harmonic Motion

A LEVEL 3

- SHM: solve with boundary conditions
- SHM with constant -boundary conditions
- SHM mixture

Second order DEs · Damped and forced oscillations

A LEVEL 10

- Complementary function: underdamped
- Complementary function: critically damped
- Complementary function: overdamped
- Solve a damped equation with ICs
- Forced (undamped): find a particular integral
- Forced (undamped): general solution
- Forced with damping - find PI
- Steady-state solution of a forced damped system
- Amplitude of the steady-state oscillation
- Damped and forced : Mixture

Coupled first-order DEs

A LEVEL 7

- Eliminate to form a 2nd equation in x
- General solution for x (distinct real roots)
- Find y from the general solution for x
- General solution for both x and y
- Particular solution - initial conditions
- Coupled system with initial conditions
- Coupled first-order DEs : Mixture

A LEVEL FURTHER MATHS

10 Numerical Integration

3 topics
12 structures

Mid-ordinate rule

A LEVEL 1

- Mid-ordinate rule

Simpson's rule

A LEVEL 3

- Simpson's rule from a table of values
- Simpson's rule for $y = f(x)$
- Simpson's rule : Mixture

Euler's method

A LEVEL 8

- One Euler step
- Two Euler steps
- Euler's method to a given x-value
- Complete an Euler table
- Euler with $\frac{dy}{dx} = f(x)$ only
- Find y_1 from given x_0, y_0 and h
- Compare with the exact value
- Euler's method : Mixture

Dimensional analysis

Dimensions and formulae — AS — 9

- State the dimensions of a quantity
- Dimensions of a product or quotient
- Find a single unknown power
- Find two unknown powers
- Find three unknown powers
- Check dimensional consistency
- Find the dimensions of a constant
- State SI base units from dimensions
- Dimensions and formulae : Mixture

Momentum and impulse

Momentum and impulse — AS — 10

- Find momentum
- Find mass or velocity from momentum
- Impulse from change in velocity
- Impulse from a constant force
- Find velocity after an impulse
- Common velocity after coalescing
- Conservation of momentum (1D collision)
- Impulse during a coalescing collision
- Average force from a bounce
- Momentum and impulse : Mixture

Collisions and restitution — AS — 10

- Direct collision: find velocities after
- Find the coefficient of restitution
- Wall impact: speed or restitution
- Successive bounces: speed
- Successive bounces: height
- Find restitution from bounce heights
- Impulse from a wall
- Kinetic energy loss in a collision
- Impulse on a particle in a collision
- Collisions and restitution : Mixture

Momentum and impulse in 2D — AS — 9

- Find momentum as a vector
- Impulse from a change in velocity
- Velocity after a vector impulse
- Magnitude of momentum
- Common velocity after coalescing
- Conservation of momentum in 2D
- Impulse during a 2D coalescing collision
- Impulse in a given direction
- Momentum and impulse :Mixture

Impulse for variable forces — AS — 8

- Impulse from $F = kt$
- Impulse from $F = at + b$
- Impulse from a triangular force-time graph
- Impulse from a trapezium force-time graph
- Speed after a variable force
- Find the constant in $F = kt$

Impulse for variable forces *continued*

- Impulse from a two-part force-time graph
- Impulse - variable forces : Mixture

Oblique Impacts — AS — 19

- Rebound speed from a smooth wall
- Rebound angle from a smooth wall
- Impulse exerted by a smooth wall
- Kinetic energy loss
- Find e from perpendicular components
- Speed after bouncing - horizontal
- Rebound angle after bouncing - horizontal
- Components of velocity after a wall impact
- Speed after successive bounces -horizontal
- Angle after successive bounces- horizontal
- Vertical component - successive bounces
- Successive impacts - 2 walls - speed
- Successive impacts - 2 walls - components
- Two walls: angle of impact
- Two walls: rebound angle
- Two walls: rebound angle - second wall
- Two walls: speed after the first impact
- Two walls: speed after both impacts
- Oblique Impacts : Mixture

Oblique collisions — A LEVEL — 12

- Rebound speed and angle from a wall
- Rebound angle from a wall
- Impulse exerted by a wall
- Loss of kinetic energy in an oblique impact
- Sphere-sphere collision: speed of B
- Sphere-sphere collision: speed of A
- Sphere-sphere collision: angle of A
- Find e from angles of incidence and rebound
- Speed after bouncing off the ground
- Rebound angle after bouncing off the ground
- Percentage kinetic energy lost
- Oblique collisions : Mixture

Work energy and power

Work — AS — 9

- Work done by a constant force
- Work done by a force at an angle
- Find μ from work against friction (incline)
- Find μ from the work done by T
- Work against friction given μ
- Work done by T given μ
- Find μ from work against friction (horizontal)
- Work done by a car engine (friction)
- Work : Mixture

Energy — AS — 17

- Kinetic energy
- Kinetic energy lost
- Increase in kinetic energy (cyclist)
- Work done - force at an angle - smooth
- Work done - force at an angle - rough

Energy *continued*

Gravitational potential energy
Speed - bottom of a smooth slope - from rest
Speed - down a smooth slope
Work-energy principle: find new speed
Energy conservation on a smooth slope (1)
Energy conservation on a smooth slope (2)
Energy conservation - resistance (1)
Energy conservation - resistance (2)
Swing: PE lost at the lowest point
Swing: maximum speed
Swing: maximum height
Energy : Mixture

Power AS 6

Power of a driving force
Average power and work done
Average power of a cyclist
Power of a winch lifting a crate
Power of a car climbing a slope
Power : Mixture

Elastic strings and springs

Hooke's law and EPE AS 18

Tension in an elastic string
Spring tension or extension
Find extension from tension
Find natural length from tension
Find the modulus of elasticity
Extension of a hanging elastic string
Minimum mass - maximum spring compression
Vertical spring equilibrium
Elastic potential energy (string)
Additional energy to stretch further
Elastic potential energy (spring)
Speed from elastic potential energy
Velocity - hanging string becomes slack
Modulus from a horizontal pull
Smooth slope: length - acceleration is zero
Smooth slope: distance when at rest
Smooth slope: speed when slack
Hooke's law and EPE : Mixture

Circular motion

Uniform circular motion AS 15

Convert between rpm and rad/s
Convert rad/s to rpm
Speed and angular speed
Period of circular motion
Acceleration from speed
Radius from speed and acceleration
Speed from radius and acceleration

Uniform circular motion *continued*

Acceleration from angular speed
Tension in a string (g, cm, rpm)
Find mass from tension
Find string length from tension
Find rpm from tension
Centripetal force from angular speed
Horizontal circle: tension or speed
Uniform circular motion : Mixture

Vertical circles A LEVEL 12

Speed at the bottom given speed at the top
Tension at the bottom
Tension at the top
Minimum speed at the top
Minimum speed at the bottom
Angle at which a particle leaves a sphere
Speed-sphere before leaving the surface
Normal reaction inside a circular track
Speed at the bottom of a circular wire
Height - particle on a string comes to rest
Greatest height - circular tube
Vertical circles : Mixture

Conical pendulum and banked tracks A LEVEL 13

Speed of a conical pendulum
Radius of a conical pendulum
Angle of a conical pendulum
Tension in a conical pendulum
Period of a conical pendulum
Length of a conical pendulum from period
Angular speed of a conical pendulum
Length of a conical pendulum from speed
Banked track angle with no friction
Maximum speed - banked track with friction
Minimum speed - banked track with friction
Normal reaction on a banked track
Pendulums - banked tracks : Mixture

Centres of mass A LEVEL 14

Centre of mass of particles on a line
Centre of mass of particles in a plane
Centre of mass from position vectors
Find mass from a given centre of mass
Centre of mass of a triangular lamina
Centre of mass of a semicircular lamina
Centre of mass of a semicircular arc
Composite lamina: two rectangles
Composite lamina: rectangle and triangle
Lamina with a rectangular piece removed
Mass needed to shift the centre
Centre of mass of an L-shaped wire
Angle of a suspended lamina to the vertical
Centres of mass : Mixture

Algorithms

Sorting algorithms · Bubble sort — AS — 10

- Bubble sort — ascending
- Bubble sort — descending
- Bubble sort — after first pass (ascending)
- Bubble sort — after first pass (descending)
- Bubble sort — number of comparisons
- Bubble sort — number of swaps (ascending)
- Bubble sort — number of swaps (descending)
- Bubble sort — comparisons in first pass
- Bubble sort — comparisons and swaps
- Bubble sort : Mixture

Sorting algorithms · Quick sort — AS — 7

- Quick sort — first pass (ascending)
- Quick sort — first pass (descending)
- Quick sort — first and second pass (ascending)
- Quick sort — first and second pass (descending)
- Quick sort — pivots in order
- Quick sort — comparisons in first pass
- Quick sort : Mixture

Bin packing — AS — 8

- First-fit algorithm
- First-fit — number of comparisons
- First-fit — number of bins
- First-fit decreasing
- First-fit decreasing — number of bins
- Full-bin strategy
- Compare FF and FFD
- Bin packing : Mixture

Order of algorithms — AS — 4

- Order from an expression
- Order from a description
- Compare orders
- Order of algorithms : Mixture

Graph Theory

Graph Theory — AS — 13

- Number of edges from a degree list
- Walk, trail or cycle
- Hamiltonian cycle or not
- Edges in a complete graph
- Edges in a complete bipartite graph
- Maximum edges in a simple graph
- Is a connected graph a tree?
- Faces and Euler's formula
- Edges in the complement
- Draw a graph from an adjacency table
- Adjacency matrix from an edge list
- Adjacency table from a graph diagram
- Graph Theory - Basics : Mixture

Algorithms · Kruskal's and Prim's — AS — 8

- Distance table from a network diagram
- Draw a network from a distance table
- Kruskal's algorithm — order of edges
- Kruskal's algorithm — order and weight
- Kruskal's algorithm from a distance table
- Prim's algorithm — order and weight
- Prim's algorithm from a distance table
- Kruskal's and Prim's : Mixture

Algorithms · Dijkstra's — AS — 5

- Dijkstra's algorithm
- Dijkstra's algorithm
- Dijkstra's algorithm - directed edges
- Dijkstra's algorithm — directed edges
- Dijkstra's : Mixture

Algorithms · Floyd's — AS — 7

- Floyd's algorithm — first iteration
- Floyd's algorithm — second iteration
- Floyd's algorithm — fifth iteration
- Floyd's algorithm — route first iteration
- Floyd's algorithm — route second iteration
- Floyd's algorithm — shortest route from a network
- Floyd's : Mixture

Algorithms · Route inspection — AS — 5

- Eulerian, semi-Eulerian or neither
- Route inspection — two odd vertices
- Route inspection — four odd vertices
- Route inspection — start and finish at different vertices
- Route inspection : Mixture

Algorithms · Travelling salesman — AS — 8

- Lower bound from a graph
- Lower bound from a distance table
- Nearest neighbour — complete graph
- Nearest neighbour — incomplete graph
- Nearest neighbour from a distance table
- Inequality from a graph
- Inequality from a table
- Travelling salesman : Mixture

Planarity algorithm — A LEVEL — 5

- Planar or non-planar
- Edges drawn outside
- Inside and outside edges
- Show a graph is non-planar
- Planarity algorithm : Mixture

Linear Programming — AS — 13

- Evaluate an objective function
- Formulate from a context
- Formulate from a context with percentages
- Is a point feasible?
- Maximise a linear objective
- Minimise a linear objective

Linear Programming *continued*

- Integer optimal point
- Integer minimum point
- Initial simplex tableau (maximise)
- Initial simplex tableau (minimise)
- Simplex method (maximise)
- Simplex method (minimise)
- Linear Programming : Mixture

Critical Path Analysis AS 14

- Activity times and the critical path
- Precedence table from an activity network
- Activity network from a precedence table
- Forward pass
- Backward pass
- Float times
- Identify the critical path
- Reducing an activity time
- Gantt chart from an activity network
- Gantt chart from a table
- Resource histogram
- Resource histogram with maximum levelling
- Two workers — minimum time
- Critical Path Analysis : Mixture

Game Theory A LEVEL 16

- Gains from a strategy pair
- Rewrite pay-off matrix for Player 2
- Stable solution and value
- Inequality for the value of a game
- Mixed strategy probability
- Value of a mixed-strategy game

Game Theory *continued*

- Remove dominated strategies
- Optimal mixed strategy
- Value of the game for Player 1
- Value of the game for Player 2
- Graphical mixed strategy for Player 1
- Graphical mixed strategy for Player 2
- Formulate as a simplex tableau
- Solve by linear programming for Player 1
- Solve by linear programming for Player 2
- Game Theory : Mixture

Binary operations and Group Theory A LEVEL 20

- Addition modulo n
- Multiplication modulo n
- Binary operations on 2×2 matrices
- Construct a Cayley table
- Commutative or associative
- Identity element
- Find the inverse modulo n
- Inverse of an element under a modular operation
- Inverse of an element from a Cayley table
- Order of a group
- Order (period) of an element
- Recognise a subgroup
- Trivial, proper and whole-group subgroups
- Group axioms from a Cayley table
- Finite and infinite groups
- Symmetries of a regular polygon
- Lagrange's theorem
- Cyclic subgroup generated by an element
- Isomorphism of finite groups
- Binary operations : Mixture

13 A LEVEL FURTHER MATHS Statistics

16 topics
177 structures

Discrete random variables

Probability distributions AS 9

- Find a missing probability
- Cumulative probability
- Double inequality
- Find the mode
- Find the median
- Probability function - find constant
- Probability function - find probability
- Probability function - inequality
- Probability distributions : Mixture

Expectation and variance (1) AS 16

- Find $E(X)$
- Find $E(X)$ (decimals)
- Find $E(X)$ from a probability function
- Find $E(X^2)$
- Find $E(X^2)$ (3 s.f.)
- Find $\mathrm{Var}(X)$
- Find $\mathrm{Var}(X)$ (3 s.f.)
- Find $\mathrm{Var}(X)$ from a probability function

Expectation and variance (1) *continued*

- Standard deviation from a table
- Standard deviation from a table (decimals)
- Expectation of a linear function (1)
- Expectation of a linear function (2)
- Variance of a linear function (1)
- Variance of a linear function (2)
- Find $E(X^2)$ ($E(X)$ and $\mathrm{Var}(X)$)
- Expectation and variance (1): Mixture

Expectation and variance (2) AS 11

- Expectation : $g(X)$
- Expectation : a/X
- Expectation : X^3
- Expectation : a/X^2
- Expectation : $aX^3 + b$
- Expectation : $aX^2 + b$ - function
- Expectation : a/X - function
- Expectation : X^3 - function
- Expectation : a/X^2 - function
- Expectation : $aX^3 + b$ - function
- Expectation and variance (2) : Mixture

Discrete uniform

AS 9

- Expectation of a discrete uniform variable
- Variance of a discrete uniform variable
- Expectation over a general range
- Calculate $P(X \leq a)$ (1-n)
- Calculate $P(X \leq a)$ (m-n)
- Calculate $P(a \leq X \leq b)$ (1-n)
- Calculate $P(a \leq X \leq b)$ (m-n)
- Calculate the standard deviation
- Discrete uniform : Mixture

Poisson distribution

Poisson probabilities

AS 13

- Find $P(X = 0)$
- Find $P(X = k)$
- Find $P(X \leq k)$
- Find $P(X \geq k)$
- Range probability for a Poisson variable
- Mean and variance of a Poisson variable
- Find t given $P(X = 0)$
- Find $P(X < \mu)$
- Find $P(X < \text{s.d.})$
- Rescaling the time period
- Poisson word problem (rescaling a rate)
- Poisson rate per hour over a clock interval
- Poisson probabilities : Mixture

Mean and variance of the binomial

AS 9

- Calculator binomial probabilities
- Mean of a binomial variable
- Variance of a binomial variable
- Mean and variance of a binomial variable
- Find $P(X < \mu)$
- Find $P(X < \mu - \sigma)$
- Find $P(X < \mu - 2\sigma)$
- Probabilities relative to the mean and s.d.
- Binomial - Mean and variance : Mixture

Sums and approximations

AS 19

- Sum of independent Poisson variables
- Poisson approximation to binomial
- Poisson approximation: cumulative
- Critical region (upper, one-tailed)
- Conclusion from a critical region upper
- Critical region (lower, one-tailed)
- Conclusion from a critical region lower
- Two-tailed critical region
- Conclusion from a two-tailed critical region
- Test using a p-value (upper)
- Test using a p-value (lower)
- Test using a p-value (two-tailed)
- Hypothesis - (upper)
- Hypothesis - (lower)
- Hypothesis - (two-tailed)
- Hypothesis test - with rescaling (upper)
- Hypothesis test - rescaling (lower)
- Hypothesis test - rescaling (two-tailed)
- Sums and approximations : Mixture

Hypothesis testing

Type I and Type II errors

AS 5

- Describe a Type I error
- Describe a Type II error
- Actual $P(\text{Type I error})$, Poisson
- $P(\text{Type II error})$, Poisson
- Type I and Type II errors : Mixture

Continuous random variables

AS 17

- Normalise a linear or quadratic pdf
- Probability from a pdf
- Find k then a probability
- Mode of a quadratic pdf
- Median of a continuous variable
- Quartiles of a continuous variable
- Quartiles from a pdf (find k first)
- $E(X)$ for a continuous variable
- $\text{Var}(X)$ for a continuous variable
- $E(aX + b)$ for a continuous variable
- $\text{Var}(aX + b)$ for a continuous variable
- Continuous uniform distribution
- Uniform probabilities
- $E(aX + b)$ $\text{Var}(aX + b)$ uniform
- Find the cumulative distribution function
- Probability from a CDF
- Continuous random variables : Mixture

Chi-squared tests

Chi-squared association

AS 10

- Look up a chi-squared critical value
- Chi-squared test statistic
- Degrees of freedom for a contingency table
- Conclusion from a chi-squared test
- Chi-squared test from a 2×2 table
- Chi-squared test with Yates' correction
- Expected frequencies from a table
- Chi-squared test from a table
- Chi-squared test with combining
- Chi-squared association : Mixture

Discrete distributions

Geometric distribution

A LEVEL 11

- Calculate $P(X = k)$
- Calculate $P(X > k)$ or $P(X \geq k)$
- Calculate $P(X < k)$ or $P(X \leq k)$
- Calculate $E(X)$
- Calculate $Var(X)$
- Find p from the mean or variance
- Calculate $P(X < a)$ or $P(X \leq a)$
- Calculate $P(X \geq a)$or $P(X > a)$
- Hypothesis test for p using a p-value
- Calculate $P(a < X \leq b)$
- Geometric distribution : Mixture

Continuous distributions

Exponential distribution

A LEVEL 15

- Calculate $P(X > t)$
- Calculate $P(X \leq t)$
- Calculate $P(a < X < b)$
- Memoryless property
- Calculate $E(X)$
- Find λ from the mean
- Calculate $Var(X)$
- Exponential waiting time-(Exp given)
- Exponential waiting time-(mean given)
- Conditional waiting time
- Conditional waiting time-(later time)
- Calculate the Median
- Find t given $P(X > t)$
- Exponential waiting time-Poisson
- Exponential distribution : Mixture

Inference

Central Limit Theorem

A LEVEL 10

- Approximate distribution - sample mean
- $E(\bar{X})$ and $Var(\bar{X})$
- Standard error of the sample mean
- $P(\bar{X} > a)$ using the CLT
- $P(a < \bar{X} < b)$ using the CLT
- Approximate distribution of the sample total
- $P(S > a)$ using the CLT
- Sample size so that $\bar{X}$ is close to μ
- CLT probability in context
- Central Limit Theorem : Mixture

Confidence intervals

A LEVEL 6

- $\mathrm{z\text{-}interval}$ for a mean with known sigma
- Margin of error for a $\mathrm{z\text{-}interval}$
- Sample size for a target margin of error
- Effect of sample size on interval width
- Width of a confidence interval
- Confidence intervals : Mixture

t-distribution and tests

A LEVEL 6

- Critical value of t
- Confidence interval - $\mathrm{t\text{-}distribution}$
- One-sample $\mathrm{t\text{-}test}$ for a mean
- Unbiased estimate of standard deviation
- Confidence interval-paired mean difference
- t-distribution and tests : Mixture

Generating functions

Probability generating functions

A LEVEL 11

- Evaluate a binomial PGF
- Recover $P(X = 0)$ from a binomial PGF
- $E(X)$ from a binomial PGF
- $Var(X)$ from a binomial PGF
- Evaluate a Poisson PGF
- Recover $P(X = 0)$ from a Poisson PGF
- Evaluate a geometric PGF
- $E(X)$ from a geometric PGF
- PGF of the sum of independent binomials
- Find $P(X = 1)$ from a binomial PGF
- Probability generating functions : Mixture