KINEMATICS

- Distance - a scalar quantity with no direction
  \[ = 160 \text{ m} \]
- Displacement - a vector quantity – measured from the starting position
  \[ = 40 \text{ m (East of starting point)} \]
- Position - a vector quantity – distance from a fixed origin

AVERAGE SPEED = \( \frac{\text{Total Distance}}{\text{Total Time}} \)

AVERAGE VELOCITY = \( \frac{\text{Displacement}}{\text{Time taken}} \)

USING Position-Time and Velocity-Time GRAPHS

Position- time graph

Velocity – time graph

VELOCITY TIME GRAPH

Gradient = acceleration

CONSTANT ACCELERATION
(Straight Line)

DISPLACEMENT – AREA UNDER THE GRAPH

VELOCITY = 0

Moving in the opposite direction – back towards the starting position

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EQUATIONS FOR CONSTANT ACCELERATION -

- s: displacement (m)
- u: initial velocity (ms\(^{-1}\))
- v: final velocity (ms\(^{-1}\))
- a: acceleration (ms\(^{-2}\))

\[
\begin{align*}
t &= \text{time (s)} \\
v &= u + at \\
v^2 &= u^2 + 2as \\
s &= \frac{1}{2}(u + v)t \\
s &= ut + \frac{1}{2}at^2 \\
s &= vt - \frac{1}{2}at^2
\end{align*}
\]

- Acceleration due to gravity is 9.8 ms\(^{-2}\) (unless given in the question)
- Negative Acceleration means retardation/deceleration
- You may need to show how the equations can be derived from the graph

A car starts from rest and reaches a speed of 15 ms\(^{-1}\) after travelling 25m with constant acceleration. Assuming the acceleration remains constant, how much further will the car travel the next 4 seconds?

\[
\begin{align*}
u &= 0 \text{ ms}\^{-1} \\
v &= 15 \text{ ms}\^{-1} \\
s &= 25 \text{ m} \\
v^2 &= u^2 + 2as \\
15^2 &= 2a \times 25 \\
a &= 4.5 \text{ ms}\^{-2} \\
u &= 15 \text{ ms}\^{-1} \\
t &= 4 \\
a &= 4.5 \\
s &= ut + \frac{1}{2}at^2 \\
s &= 15 \times 4 + \frac{1}{2} \times 4.5 \times 16 \\
&= 96 \text{ m}
\end{align*}
\]

A ball is thrown vertically upwards with a speed of 12 ms\(^{-1}\) from a height of 1.5 m. Calculate the maximum height reached by the ball.

\[
\begin{align*}
u &= 12 \text{ ms}\^{-1} \\
a &= -9.8 \text{ ms}\^{-2} \\
\text{At maximum height } v &= 0 \\
v^2 &= u^2 + 2as \\
0 &= 144 - 2\times9.8\times s \\
s &= 7.35 \text{ m} \\
\text{Maximum height} &= 1.5 + 7.35 \\
&= 8.85 \text{ m}
\end{align*}
\]

2 FORCES and ASSUMPTIONS

KEY FORCES
- W: weight (mg = mass \times 9.8)
- R: reaction (normal reaction – at right angles to the point of contact)
- F: friction (acts in a direction opposite to that in which the object is moving or is on the point of moving)
- T: Tension

ASSUMPTIONS
- Objects are modelled as masses concentrated at a single point so no rotational forces.
- Strings are inextensible (inelastic) so any stretch can be disregarded
- Strings and rods are light (no mass) so weight can be disregarded
- Pulleys are smooth so no frictional force at the pulley needs to be considered.
RESOLVING FORCES

Calculate the resultant force acting on the particle giving you answer in the form $ai + bj$

**Working in i (horizontal)**
- $RI = 35\cos 45^\circ - 12$
- $= 12.7\ \text{N}$

**Working in j (vertical)**
- $RJ = 35\sin 45^\circ - 10$
- $= 14.7\ \text{N}$

$$R = 12.7\ \text{i} + 14.7\ \text{j}$$

The forces shown in the diagram act on particle A of mass 0.8 kg. Calculate the magnitude of the acceleration of the particle

Resultant force $= 12.7i + 14.7j$

Force $= \text{mass} \times \text{acceleration}$

$0.8a = 12.7i + 14.7j$

$a = 15.875i + 18.375j$

$$|a| = \sqrt{15.875^2 + 18.375^2}$$

$$|a| = 24.3\ \text{ms}^{-2}\ (3\ \text{s.f.})$$

If the system is in **Equilibrium**, then resultant force $= 0$

Take care with objects on slope – always draw a diagram showing all the forces

A box of mass 5 kg rests on a slope inclined at an angle of $30^\circ$ to the horizontal

Calculate the normal reaction and the friction

**Resolving perpendicular to the slope**
- $R = 5g \cos 30^\circ$
- $= 42.4\ \text{N}\ (3\ \text{s.f})$

**Resolving parallel to the slope**
- $F = 5g \sin 30^\circ$
- $= 24.5\ \text{N}\ (3\ \text{s.f})$

COEFFICIENT OF FRICTION

The maximum or limiting value of friction $F_{\text{max}}$ is given by

$$F_{\text{max}} = \mu R$$

$R$ is the normal reaction and $\mu$ is the **coefficient of friction**

If a force is acting on the object but the object remains at rest then $F < \mu R$

When the object is moving the frictional force is constant ($F_{\text{max}}$)

For questions looking at the minimum and maximum force needed to move a block on a rough slope look at the magnitude of force $P$

When the block is on the verge of sliding down the slope

Friction is ‘acting up the slope’

When the block is on the verge of sliding up the slope

Friction is ‘acting down the slope’
NEWTONS LAWS

1st LAW: Every object remains at rest or moves with constant velocity unless an external force is applied.

The system is in EQUILIBRIUM

\[ T = F \]
\[ R = W \]

2nd LAW: The resultant force acting on an object is equal to the acceleration of that body times its mass.

\[ F = ma \]

3rd LAW: If an object A exerts a force on object B, then object B must exert a force of equal magnitude and opposite direction back on object A.

Calculate the acceleration of the object
Resultant force = 12000 – 6000
= 6000 N
6000 = 8000a  \(a = 0.75\) ms\(^{-2}\)

A man of mass 80 kg stands in a lift
Calculate the normal reaction of the lift floor on the man if
a) The lift is moving downwards with constant velocity
   Constant velocity so R = W
   R = 80g N
   = 784 N
b) The lift is moving upwards with acceleration of 2 ms\(^{-2}\)
   Upwards movement so R > W  \(R - 80g = 80\times 2\)
   \(R = 944\) N

Two masses are connected by a light string passing over a smooth pulley as shown below. Calculate the acceleration of the 4 kg block when released from rest.

5 kg Block : 5g – T = 5a
4 kg Block :  \(T – 2 = 4a\)
Solving simultaneously  5g – 2 = 9a
\(a = 5.22\) ms\(^{-2}\)

Forces \(F_1 = 2i + j\), \(F_2 = -3i + 4j\) and \(F_3 = 4i – 6j\) act on a particle with mass 10 kg. Find the magnitude of acceleration of the particle

Resultant force = \(F_1 + F_2 + F_3 = (2i + j) + (-3i + 4j) + (4i – 6j)\)
= \(3i – j\)
\[F = ma\]
\(3i – j = 10a\  \ a = 0.3i -0.1j\  \ |a| = \sqrt{0.3^2 + (-0.1)^2}  \ a = 0.316\) ms\(^{-2}\)
The acceleration of a particle (in \(\text{ms}^{-2}\)) at time \(t\) seconds is given by \(a = 12 - 2t\).
The particle has an initial velocity of \(3\ \text{ms}^{-1}\) when it starts at the origin.

a) Find the velocity of the particle after \(t\) seconds
\[
v = \int 12 - 2t \, dt
\]
\[
v = 12t - t^2 + c \quad t = 0 \quad v = 3 \quad c = 3
\]
\[
v = 12t - t^2 + 3
\]

b) Find the position of the particle after \(t\) seconds
\[
r = \int (12t - t^2 + 3) \, dt
\]
\[
r = 6t^2 - \frac{t^3}{3} + 3t + c
\]
\[
r = 0 \quad t = 0 \quad r = 6t^2 - \frac{t^3}{3} + 3t
\]

A train moves between 2 stations, stopping at both of them
Its speed at \(t\) seconds is modelled by \(V = \frac{1}{5000} t(1200 - t)\) (\(\text{ms}^{-1}\))

Find the distance between the 2 stations

At the stations \(v = 0\)
\[
\frac{1}{5000} t(1200 - t) = 0 \quad t = 0 \quad t = 1200
\]

Distance = \(\int_0^{1200} \frac{1}{5000} t(1200 - t) \, dt\)
\[
= \frac{1}{5000} \left[ 600t^2 - \frac{t^3}{3} + c \right]
\]
\[
= 57600 \text{ m}
\]
\[
= 57.6 \text{ km}
\]
### PROJECTILES

**Initial Velocity** : \( u = U \cos \theta \, \mathbf{i} + U \sin \theta \, \mathbf{j} \)

**Acceleration** : \( a = -g \, \mathbf{j} \)

**Velocity after t seconds**:
\[ v = U \cos \theta \, \mathbf{i} + (U \sin \theta - gt) \, \mathbf{j} \]

Particle moving in a horizontal direction (reaches **maximum height**) when \( \mathbf{j} \) component = 0 \hspace{1cm} U \sin \theta - gt = 0

**Displacement after t seconds (r)**
\[ R = U \cos \theta \, \mathbf{i} + (U t \sin \theta - \frac{gt^2}{2}) \, \mathbf{j} \]

If launched from the ground the particle will return to the ground when \( r_j = 0 \hspace{1cm} U t \sin \theta - \frac{gt^2}{2} = 0 \)

Solve the equation to find the values of \( t \). Substitute ‘t’ into the \( \mathbf{i} \) component of \( r \) to calculate the **range**

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### MOMENTS

The moment of a force about point A is

**moment** = **Force** \( \times \) **d** \hspace{1cm} (Nm) \hspace{1cm} (anticlockwise)

The moment of a force about point B is

**moment** = **Force** \( \times \) **dsin \theta** \hspace{1cm} (Nm) \hspace{1cm} (clockwise)

\( d \) is the perpendicular distance of the line of action of the force from A

The **resultant moment** is the difference between the sum of the clockwise moments and sum of the anticlockwise moments (in the direction of the larger sum)

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A shot putter releases a shot at a height of 2.5m, with speed 10 ms\(^{-1}\) at an angle of 50\(^\circ\) to the horizontal. Calculate the horizontal distance from the thrower to where the shot lands.

\( u = 10 \cos 50^\circ \, \mathbf{i} + 10 \sin 50^\circ \, \mathbf{j} \)

\( a = -9.8 \, \text{ms}^{-2} \, \mathbf{j} \)

\( s = 10 \cos 50^\circ \, \mathbf{i} + (10 \sin 50^\circ - 4.9 t^2) \, \mathbf{j} \)

Starting height = 2.5 m so hits the ground when \( s_j = -2.5 \)

\( 10 \sin 50^\circ - 4.9 t^2 = -2.5 \)

\( t = -0.277 \) or \( t = 1.84 \)

Horizontal distance travelled when \( t = 1.84 \)

\[ = 10 \times 1.84 \cos 50^\circ \]

\[ = 11.8 \text{ m (3 s.f.)} \]
Uniform Lamina - (usually rectangular) – has same density throughout – Centre of mass through which the objects weight acts is the centre of the rectangle

Uniform Rod centre of mass is at the midpoint of the rod

Equilibrium
If an object is in equilibrium the resultant force is zero and the total moment of all the forces is zero
To solve problems
- Draw a diagram showing all the forces
- By taking moments about a point you can ignore the forces acting at that point (d = 0)
- Resolve the forces horizontally, vertically

A uniform bridge across a steam is 5m long and has a mass of 100 kg. It is supported at the ends A and B. A child of mass 45 kg is standing on the bridge at point C. Given that the magnitude of the force exerted by the support at A is three-quarters of the magnitude of the force exerted by the support at B calculate the magnitude of the force exerted at support A and the distance AC

\[
\begin{align*}
\text{Resolving vertically: } 1.75R &= 145g \\
R &= 82.9\text{g N (3 s.f.)} \\
\text{Force at A: } 0.75R &= 62.1\text{g N} \\
\text{Taking moments about A: } (2.5 \times 100) + (45 \times d) &= 5 \times 82.9 \\
250 + 45d &= 414.5 \\
d &= 3.66 \text{ m}
\end{align*}
\]

NOT ON ALL EXAM BOARDS
A uniform ladder of length 3 m, and mass 20 kg, leans against a smooth, vertical wall, so that the angle between the horizontal ground and the ladder is 60°. Find the magnitude of the friction and the normal reaction forces that act on the ladder if it is in equilibrium

Step 1: Draw a diagram showing all of the forces

Step 2: Resolving Vertically
\[
R_2 = 20g \quad R_2 = 196\text{ N}
\]

Step 3: Taking moments at the base
\[
20g \times 1.5 \cos 60° = R_1 \times 3 \sin 60° \\
R_1 = 56.6 \text{ N}
\]

Step 4: Resolving horizontally
\[
F = R_1 = 56.6 \text{ N}
\]